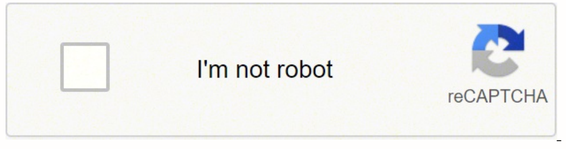

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Some problems such as the analysis of load combinations cannot even be formulated without recourse to probabilistic reasoning. When the loading process is continuous then the probability distribution of the maximum value (largest extreme) is likely to be very closely approximated by one of the asymptotic extreme value distributions, treated in section 3.3. In this way instead of modelling a single load variable as a stochastic process {X(t)} it is modelled by a stochastic variable, say Y (see also section 9.5). Div., ASCE, Vol. The first stage of this process is to decide upon an appropriate standard of reliability or target failure probability for the structures (or more generally, structural components, e.g. beams, columns, slabs) that will be designed using the new code. C.: Structural Accidents and their Causes. [9.3) Lin, Y. 66, 1969, pp. The load-carrying capacity of the column may be assumed to be governed by the relationship 76 4. Checks on the consistency of the means and variances of the various subsamples (see for example [3.51) should generally be undertaken when practicable. 107 6.3 CORRELATED BASIC VARIABLES Example 6.7. Consider again the beam shown in figure 6.5, but now only c and 2 are considered realizations of random variables. Figure 7.5 is useful in calculating the distribution function FR for the strength R of the series system. The target failure probability Pft for the new code BS 5400: Part 3 was determined as the weighted average of the failure probabilities for components designed to BS 153 and was 0.63 X 10-6. , f.i n). Arnold, London, 1966. & Dunsmore, I. are the (unknown) I n. The expected rate of positive crossings of any barequal to rier can then be calculated from (9.35). The cumulative frequency diagram is therefore obtained by plotting the points (xi' i/(n + 1» using scales appropriate to the type of distribution function. Chapter 12 on offshore structures should be of interest to those working in this field. Whilst many important contributions to the literature are thus omitted, it is considered that this selective approach will be of more help to the new reader. The values of J..IR and J..IR given above are typical for the yield stress of grade 50 and 43 weldable structural steels, and it can therefOJ;e be seen that a gross error involving the substitution of grade 43 steel for grade 50 in a critical part of a structure is quite likely to cause failure - a chance of about 1 in 6 with the arbitrary assumptions made here. It will take the form I n (3.33) Pi =1 1 where Pi is the probability that a bar will be supplied by manufacturer i and where fX/Al (x) = ql fX/Al n Bl (x) + q2fx /A 1 n B2 (x)+ ... and Moskowitz, L.: A Proposed Spectral Form for Fully Developed Wind Seas based on the Similarity Theory of S. Figure 12.11. Report No. 63, 1977. Exercise 2.9. Consider two jointly distributed discrete random variables Xl and X2 with the probability mass functions PX 1 and PX 2 given in figure 2.13 and assume that Xl and X 2 are i~dependent. Using equation (3.46) it can be shown that tl = E[Xj = IJ. [2.7] Larson, H. The concept of autocorrelation is discussed in chapter 9. Example 4.1. Assume that some delicate instrument is destroyed by horizontal accelera· tions greater than 0.05g and that the time interval between major earthquakes can be assumed to be exponentially distributed. To be more precise, he wishes to know the probability distribution of the greatest load. [12.33] Rice, S. The need for a number of design checks using different sets of lji 0 factors arises from the fact that throughout a structure the contribution of each separate load Qj to the maximum loadeffect in any member, varies considerably from member to member. 39-73. As discussed in chapter 3, statistical data may be available to quantify some of the sources of uncertainty and this information can be included in a suitable form. Bell Systems Tech. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use. In such cases the actual strength, and hence the reliability, is not a continuous function of the partial coefficients. Analytical solutions do not exist for the majority of practical problems and the analyst must resort to numerical methods. Structures or structural components »faii» when they encounter an extreme load, or when a combination of loads causes an extreme load effect of sufficient magnitude for the structure to attain a))failure state)); this may be an ultimate or a serviceability condition. R., O'Brien, M. It is usual to order the elements of the sample (xl' x 2 ' •.. Then (4.45) As a general rule, however, Monte-Carlo methods should be avoided if at all possible. [3.9] Joint Committee on Structural Safety, CEB - CECM - CIB - FIP - IABSE - lASS - RILEM: General Principles on Reliability for Structural Design. This is known as the central limit theorem. In the latter case, the engineer or team is clearly responsible for the failure. 4.3.2 The Fundamental Case For a simple structural member selected at random from a population with a known distribution function FR of ultimate strength R in some specified mode of failure, the probability of failure Pf under the action of a single known load effect s is Pf = peR - s';;; 0) = FR (s) = P(R/s';;; 1) (4.13) If the load effect S is also a random variable, with distribution function Fs , equation (4.13) is replaced by (4.14) 72 4. A series system with n elements is generally symbolised as shown in figure 7.5. All elements in figure 7.5 are brittle elements but for a series system the distinction between brittle and ductile elements is irrelevant because the total system fails as soon as one element fails whether it is brittle or ductile. The important equation (6.5) could therefore be used in these examples. The failure types are defined as ductile failure with reserve strength capacity resulting from strain hardening II ductile failure with no reserve capacity III brittle failure and instability Having chosen a target failure probability, the problem of selecting a set of partial coefficients r for a code, or part of a code, may now be reduced to the application of the following simple principle. The value of the diameter 0* which will provide this reliability is found as follows: ~ = .p-1 (0.9999) = + 3.72 (from tables of the standard normal distribution function), and since Qn8 is normally distributed Qn 0* = !1 Qne + 3.72o Qne = 4.7646 giving 0* = 0 = exp(4.7646) = 117.3 mm. This should not be confused with the concept of optimizing individual structures. It can be shown that if and a = (8t ± 3) where band t are integers, then the length of the integer sequence before repetition is of the orderof2(b-2). Early designers cannot be criticised for not knowing about such effects, just as there is no reason to suppose that currently unrecognised failure modes will not cause accidents in the future. As shown in example 5.2 the safety margin M = R - S results in the reliability index !lR -!ls (o~ + (5.24) 1 0 8)2 By using the equivalent safety margin (5.8) M = Qn(RjS) = QnR - QnS one gets {J' = /.lQn(R(S) (5.25) o~n(R(S) Exercise 5.2. Show that the following approximate value of {J' defined by (5.25) Qn/.l R - Qn/.ls JtR)2 + t)2 /.lR (5.26) S !ls is obtained by linearization of the safety margin M = Qn(RjS) about (/.lR' /.lS). The reliability index {J as defined by equation (5.9) will change when different but equivalent non-linear failure functions are used. A. For a particular structure and failure mode, the sensitivity factors Ii may be evaluated from equation (11.21). It can be used as a measure of mutual linear dependence between a pair ofrandom variables. II. K.: Strength- and Load Processes. LOAD COMBINATIONS 162 o T T Figure 10.1 The intention of chapter 10 is to give some information on problems connected with load combinations. It should be noted that the preceding classification applies both to the actions themselves and to the mathematical models that are used to represent them. J.: Statistics of Extremes. W Wx the failure surface in the x- The formal definition (6 .20) is convenient if a computer is available, because calculation of ~ is by (6 .20) formulated as a constrained optimization problem. This is because manufacturers with high product variability have to aim for higher mean properties than manufacturers whose products can be closely controlled to achieve the same specified yield stress, for a given probability ofrejection. For present purposes, only results for a single tubular member are discussed (indicated by the arrow in figure 12.11). 189, 1950. and & for the parameters Jl. and a of a normal distribution. A number of examples in chapters 5 and 6 of analysis and design of simple structures loaded by single loads illustrate this fact. The approach was first used and has subsequently been further developed by Lieblein [3.10] for extreme-value distributions. The subject is now sufficiently well developed for it to be included as a formal part of the training of all civil and structural engineers, both at undergraduate and post-graduate levels. The values {J and {J' from (5.24) and (5.26) are different. If the standards of quality control are significantly different between different manufacturers or suppliers (e.g. in the case of steel or concrete), it may be convenient to use a mixed distribution model to allow for these differences. For any particular action and structure, the attributes listed above also govern the nature of the structural analysis that must be undertaken. The problem is to know the various forms which gross errors can take. In both cases, it is the sum of the horizontal squared deviations from the line which should be minimised, not the vertical (assuming the axes are chosen as shown in figure 3.4). 11.3.5 Treatment of loads and other actions The classification and modelling of loads and other actions were discussed in chapter 3. E. Journal of Geophysical Research, Vol. The failure probabilities exhibit very wide scatter varying over many orders of magnitude. and a. The modelling of combinations of loads is discussed in chapter 10. The correlation between P1 and P2 is given by Calculate the reliability index B when the following failure criterion is used u max ::;, P 1 100 Q In examples 6.5 - 6.6 and exercise 6.3 the correlation between random variables could be treated in a simple way because these random variables only appeared in a linear connection. In section 3.5, it is also shown that when dealing with a single time-vary- ing load in connection with barrier crossing problems (see section 9.4) the detailed time variation is not of relevance. © Springer-Verlag Berlin, Heidelberg 1982 Softcover reprint of the hardcover 1st edition 1982 The use of registered names, trademarks, etc. Show that the marginal density functions fXl (xl) for (2.87) are 1 fXl(X I)= f2V L.1f ul 1 Xl-Ill 2 exp[-"2(U) 1 I (2.88) 35 BIBLIOGRAPHY The multivariate normal density function is defined as 1 1 1 n f-(x) = - - . Show that n n E[L'fj(Xj)] = IE[fj(X j)] j=l j=l (2.77) so that the operation of expectations and summations can commutate. 114 7. APPLICATIONS TO STRUCTURAL CODES aged for use in level 1 codes. The example given above shows how Rand S can be modelled by a number of component variables. The type II maxima distribution is frequently used in modelling extreme hydrological and meterological events. Some further results using a more sophisticated shell analysis for the stiffened cylindrical leg members, in place of the API failure criterion, have been reported in [12.2). Draft for Development, DD55: 1978. A final effect which must be taken into account is the systematic change in mean yield stress with bar diameter as illustrated in figure 3.9. This phenomenon is quite marked and is rarely taken into account in structural design. First a set of uncorrelated basic variables X= (Xl, ... For this type of structural configuration (in fact, a parallel ductile structural system in the reliability sense - see chapter 7) in which the structural strength is governed by the average strength of the components, it can be anticipated from the above - although it will not be formally proved here - that the reliability of the structure is not sensitive to the extreme lower tails of the strength distributions of the components. Baker Structural Reliability Theory and Its Applications With 107 Figures Springer-Verlag Berlin Heidelberg New York 1982 PALLE THOFT-CHRISTENSEN, Professor, Ph. D. ,n are used. The third step of evaluating suitable distribution parameters is discussed in section 3.6. Permanent loads: The total permanent load that has to be supported by a structure is generally the sum of the self-weights of many individual structural elements and other parts. Dover, N. This can be done either directly, by modifying some part of the structure, or indirectly by modifying the partial coefficients. Eng. Safety and Reliability of Eng. These distributions were discussed in chapter 3. Whether or not a formal optimization is undertaken in practice, it is useful to think of the partial coefficient selection process in this way. [6.6] The Nordic Committee on Building Regulations (NKB): The Loading and Safety Group, Recommendations for Loading and Safety Regulations for Structural Design. , n are equal to the eigenvalues of C Example 6.3. Consider two correlated random variables Xl and X 2 with the mean vector and the covariance matrix 100 6. Considerable success has been achieved in the analysis of structural failure data using this type of classification [13.8], [13.11], [13.17], [13.18]. 35, 1981. Failure occurs when M' < 1, or when M < O. , n and variance and covariance collected in the covariance matrix by equation (2.84)) Co,IX" X, I .... 16 mm plates /- / /- N/m 260 240 220 320 300 280 Figure 3.10. It has been shown in the literature (see the references at the end of chapter 10) that the upper bound (10.12) is very close to the exact result, so that it can be used as an approximation for !Ix (0. & Castanheta, M.: Structural Safety. A further distinction that should be made is between loading models used for the purposes of normal (deterministic) design and those required for structural reliability analysis. Sydney, 1979. 27 0 .16 + 27 . It will be shown that it is often important to take account of this correlation. An important conclusion is that many gross errors occur because of lack of experience on the part of those undertaking the work and because the fundamental behaviour of the structure is often not fullymderstood. -;::- y ~ / ,/ ~ ~ ,/ ~ 0.05 0.02 0.01 0.005 220 I J Ie 240 N/mm 260 280 300 320 340 360 Figure 3.11. n X n 3.3 ASYMPTOTIC EXTREME-VALUE DISTRIBUTIONS It is fortunate that for a very wide class of parent distributions, the distribution functions of the maximum or minimum values of large random samples taken from the parent distribution tend towards certain limiting distributions as the sample becomes large. I.: The Nature of Structural Design and Safety. , xn) and f x(i) is the probability density function for the random vector X. However, as mentioned in section 11.3.2, the user of a level 1 code may often not know the actual characteristic values for his material properties in advance, and it is generally necessary to design using specified characteristic values, esp ' The acceptance criteria for a material should be devised so that ek exceeds e sp at a stated level of probability Pe' It should be noted here that the uncertainty associated with the event (e k > esp) arises as a result of imperfect knowledge of the ma- terial supplied and the difficulties of obtaining sufficient sample data at the appropriate time. 2nd International Conference on the Behaviour of Off-Shore Structures, London, 1979. , Xn) then approximate values for can be obtained by using a linearized safety margin M. For detailed information on this subject the reader should refer to a specialist text, e.g. Gumbel [3.8) or Mann, Schafer and Singpurwalla [3.111. Example 5.3. Consider the statically indeterminate beam shown in figure 5.3 loaded by a concentrated force p and assume that the beam fails when Iml;;' m F , where m F is a critical limit moment and m is the maximum moment in the beam. [3.13] Mayne, J. A reduction in the number of partial coefficients can be achieved by constraining the unwanted coefficients to be unity. = c8,H.. IABSE. [12.9) British Standards Institution: Fixed Offshore Structures. Uncertainties exist in most areas of civil and structural engineeri'1.g and rational design decisions cannot be made without modelling them and taking them into account. [3.6] Edlund, B. Construction Industry Research and Information Association, Report 25, London, 1970. When the basic variables X = (Xl, ... This means that they must be applicable over a range of sensitivity factors without being unsafe or unduly conservative. EXTENDED LEVEL 2 METHODS Exercise 6.3. Consider the beam shown in figure 6.5 loaded with two concentrated loads Pl and P2 at the same point on the beam. 1976. APPLICATIONS TO STRUCTURAL CODES Clearly, many other possibilities exist. Wiley & Sons, N.Y., Vol. 11.5.1 Aims of calibration BS 5400: Part 3 is a level 1 code in which the degree of structural reliability is con tolled by a number of partial coefficients (partial factors). and Longuet-Higgins, M. For each variable, the characteristic value ek should be such that it has a reasonably high probability q (= 1 - p) of being exceeded in any single trial or test. ACI-Journ., Vol. Co,IX" Xoi Cov[X n , X 2] . MULTIPLICATIVE CONGRUENCE METHOD This method produces a series of pseudo-random numbers rj that eventually repeats, but, if correctly designed, only after a very long cycle. The process of selecting the set of partial coefficients to be used in a particular code should be seen as a process of optimization such that the outcome of all designs undertaken to the code is in some sense optimal. The failure probability may be estimated in at least two ways. Let us first examine the role of partial coefficients. They will then be researched and this will add to the general fund of engineering knowledge. As mentioned above, loads and resistance variables require different treatment and will be discussed separately. The penalty to be paid for increased simplicity in the code safety format is therefore some increase in the initial cost of construction. Paper 2192, Houston, 1975. Holden-Day, San Francisco, 1969. It will now be shown how a (new) set of random variables Y = (Y l , ... McGraw-Hill, N.Y., 1967. 71 4.3 STRUCTURAL RELIABILITY ANALYSIS In contrast to electronic/mechanical systems, structural systems tend not to deteriorate, except by the mechanisms of corrosion and fatigue, and in some cases may even get stronger for example: the increase in the strength of concrete with time, and the increase in the strength of soils as a result of consolidation. cathodic protection system becomes inoperative Table 13.1. General classification of the nature and sources of gross errors. 69, No. 24, Dec. 53 - 72. Control takes two main forms • quality control of materials and fabrication, and • controls to avoid the occurrence of major or gross errors in the design and construction processes. m.1 ="" (x.)l n":::" I i=l (3.48) Finally, the moment estimators OJ, j = 1, ... The process is interrupted approximately every 600 m because the continuously cast steel is cut into ingots and these are re·heated and rolled separately. The maximum deflection is 1 pcQ 2 9y'3 ei c ~----------------~,~v--------~k Figure 6.5 where e is the modulus of elasticity, i the relevant moment of inertia and p = Pl + P2' Further, let p, c, Q, e and i be realizations of random variables P = P1 + P2 , C, L, E and I with E[P1] E[P2] E[C] 2m Uc Om E[L] 4m uL Om E[E] 4,10-5 m-4 uE 0.3' 10-5 m 4 E[I] 4'10 4 MN/m 2 ur = 8 kN up! = uP2 = = 0.32 kN 104MN/m2 All random variables except P1 and P2 are assumed uncorrelated. Report UR8, CIRIA Underwater Engineering Group, 2nd edition, Aug. and Oit = Var[M] = O~ + O~ (4.24) giving 1 oM = (02 + 02)2 RS (4.25) Since Rand S are normal, 1\1, a linear function of Rand S, is also normally distributed and (M - J.lM)/oM is unit standard normal, giving P =(O-J.lM)= (J.lS-J.lR) C oM 1. In addition, each limit state function is likely to involve one or more constants c. [13.10] Matousek, M.: Massnahmen gegen Fehler im Bauprozess. e FX (x) = c(x - e)k (3.21) i.e. the parent distribution is limited to the left at a value x = e. K.: Probabilistic Theory of Structural Dynamics. Frequently, time to failure data are available for the individual components, but are not available for the complete system. 5) Evaluate {3 from equation (12.79). (See chapter 9 for further details of stochastic processes). Fiessler: An Algorithm for Calculation of Structural Reliability under Combined Loading. The actual characteristic value of the imperfection ik can conveniently be chosen as the 95% fractile of I and the acceptance criteria designed so that isp exceeds ik by an appropriate margin (or with a stated probability) - see figure 11.1 (b). The main reason is that only by using partial coefficients can reasonably consistent standards of reliability be achieved over a range of different designs within anyone code. [9.5) Papoulis, A.: Probability, Random Variables and Stochastic Processes. ,!In) is chosen, but as shown later, a point on the failure surface would be more reasonable. Some typical data giving values of Uyd for consecutive lengths of 20 mm diameter hot-rolled high-yield bars from the same cast of steel are shown in figure 3.8 (along with values for the static yield stress). In this the real structure is modelled by an equivalent system in such a way that all relevant failure modes can be treated. Example 3.1. For i = n equation (3.4) simplifies to: (3.5) FX n (x) = (Fx (x»n n This is the distribution function for the maximum value in a sample size n. 192 11. It is a subject which has grown rapidly during the last decade and has evolved from being a topic for academic research to a set of well-developed or develop ing methodologies with a wide range of practical applications. 183 11.3. RECOMMENDED SAFETY FORMATS FOR LEVEL 1 CODES The characteristic value x k of a basic random variable X is defined as the pth fractile of X given by (11.7) where F~ is the inverse distribution function of X, and p is a probability which depends on the type of variable being considered (i.e. a load or a strength). EXTENDED LEVEL 2 METHODS c-X = [: :1 The characteristic equation of Cx is with the roots Al = 2 and A2 = 4. APPLICATIONS TO STRUCTURAL CODES Having estimated the sensitivity factors (i from equations (11.29) to (11.35), the partial coefficients 'Yi and 'Yi' or the design values of the variables xt and xj, may be obtained directly from equations (11.25) and (11.26). BIBLIOGRAPHY [12.37] 237 Van der Hoven, L:Power Spectrum of Horizontal Wind Speed in the Frequency Range from 0.0007 to 900 Cycles per Hour. 11.3.2 Characteristic values of basic variables The term characteristic value was introduced in the late 1950's at the time when probabilistic concepts were first being introduced into structural codes; and when it was recognised that few basic variables have clearly defined upper or lower limits that can sensibly be used in design. Laboratorio Nacional de Engenharia Civil, Lisbon, 1971. In this, the target failure probability depends on the consequences of failure and on the nature of the failure mode, as shown in table 11.3. Failure type Failure consequences Not serious Serious Very serious III II 10-4 10-3 3.09 5 4.26 3.71 4.26 4.26 10-6 10- 10-4 10-5 10-5 3.71 10-6 4.75 4.75 10-7 5.20 Table 11.3. Target failure probabilities and corresponding reliability indices [11.10]. However, upper and lower bounds for !lx(O can be derived by changing the domain of integration (w) in (10.11) in an appropriate way. M.: Safety, Reliability and Structural Design. The failure function can be written as M = 16 z E X P Y m - Q2 (G + Q) X 106 = 0 (11.37) Using the methods of chapter 5 and the parameters from table 11.1 gives a reliability in· dex ~ = 4.45. EXTENDED LEVEL 2 METHODS 104 It is easy to see that the reliability index!3 defined by equation (6.20) can be evaluated on the basis of a safety margin, when the failure function is linear in the basic variable. There are some circumstances, however, when increases in rQ or in rm may not give rise to these effects. In the following all stochastic processes will have the same index set so that the shorter notations {Xl (t)}, {X 2(t)}, etc. If uyd is plotted against Z, the position in the bar, the outcome will be of the form shown in figure 3.7. This is an example of a step-wise continuous-state/continuo us-time stochastic process X(t) in which the parameter t may be interpreted as the distance Z along the reinforcing bar. 100 mm slab instead of 150 mm Design and analysis Construc- - Use of incorrect material · .. A further aspect of modelling must now be introduced. This relationship between p and ~/~* is shown in figure 6.4. {Ji{3 * ----~----+_----+_--·-4I----_+----~>_p Figure 6.4 106 6. A detailed discussion of this subject is beyond the scope of this book. [4.3] Barlow, R. The reliability function !itT which is the probability that the system will still be operational at time t is given by !ltT(t) = 1 - FT(t) = 1 - P(T .;;; t) = P(T > t) (4.1) If the density function fT of the time to failure is known then !itT may be expressed as (4.2) In some circumstances there are good a priori reasons for selecting a particular form for fT for example, a Weibull distribution, whose distribution function takes the form (4.3) Substituting in equation (4.1) gives t;;;. NKB-Report No. 36, November 1978. A similar problem of lack of invariance arises when the partial coefficients used in a code are not directly associated with their corresponding sources of uncertainty. The former is a fractile of a random variable, whereas the latter is some specified single value of the same quantity - a constant. Then 1 x ='Y - ex £n(- £nFX (x» (A.9) 252 APPENDIX A. Such an approach has many practical advantages. The covariance Cov[QBMB] can be evaluated by (6.1), where 97 6.2 CONCEPT OF CORRELATION t ~PI z A A ~ /3 Q13 • ~ ~ Q "It ~(~ B Q/3 •v = 6 m •" Figure 6.1. 1 6 E[QBMBI = E[27 (13 PI + 23 P 2) 27 (4 PI + 5 P 2)1 = 2~2 (52 E[Pil + 157 E[P 1 P21 + 115 E[P~1) In this equation E[P~ I = a~1 + f..l~1 = 0.16 + 16 = 16.16 (kN)2 E[P~l = a~2 + f..l~2 = 0.25 + 36 = 36.25 (kN)2 Therefore E[QBMB1 = 2~2(52.16.16 + 157·24 + 115·36.25) = 72.239 (kN)2m and Finally, the correlation coefficient is PQB MB 0.3051 = 0.467.0.660 = 0.99 This value close to one indicates that Q B and MB are strongly correlated. Exercise 6.1. Consider the cantilever beam AB shown in figure 6.2, loaded with two statistically independent random loads P 1 and P 2 ' where f..lPI = f..lP2 = 5 kN and a pI = ap2 = 1 kN. Example 6.1. Consider the beam AB shown in figure 6.1 loaded with two random loads P I and P2' The loads are assumed statistically independent with means J1P! = 4 kN and J1P 2 = 6 kN and standard deviations ap! = 0.4 kN and ap 2 = 0.5 kN. , Yn), where Yi , i = 1, 2, ... Its density function is given in equations (2.55) and (2.56). 111 -121. This approach should not be neglected in any serious application of these methods. In general terms, the aim Of this approach is to minimise the deviations of the probabilities Pfi from the target probability of failure Pft' whilst maintaining the average probability of failure at the target level. RANDOM NUMBER GENERATORS Hence, a sequence of independent random deviates xi of the random variable X may be obtained from 1 x.1 = r - £n(- £nr.) Q 1 (A.10) where r i are rectangularly distributed random numbers in the interval [0, 11. , Xn)' that is Y = f(X) = f(X I ,·⋯, Xn) (2.75) 33 2.9 FUNCTIONS OF RANDOM VARIABLES It can be shown that (2.76) where i = (xl' ... Proc. Dissertation No. 6941. MODAL ANALYSIS The matrix equation (B.8) represents a set of coupled differential equations, which can be uncoupled by using the system's eigenvector matrix (mode shape matrix), if, as a transformation matrix. The pseudo-random numbers rj generated by this method can be assumed to be independent realisations u j of a random variable U having a rectangular distribution with 0 ,;;;; U';;;; 1. 186 11. It should be noted that in all cases, the questions to be answered are of the type: Is the structure strong enough? 4_ GENERATION OF RANDOM DEVIATES WITH A SPECIFIED PROBABILITY DISTRIBUTION FUNCTION Fx A convenient general method consists of generating a random number, r, as described above and then, by making use of equation (A.2), finding the corresponding random deviate x of the random variable X from (A.5) where Fx is the required distribution function. [5.4] Dyrbye, C. Ferry Borges. 82, May 1977. In this chapter the transformation given by Rackwitz and Fiessler [6.4] will be used. The role of quality assurance, in its broadest sense, is to coordinate, rationalise and monitor these two stages. [12.18) Hohenbichler, M. What are of interest, however, are the parameters Qi' which indicate the relative sensitivity of the failure probability to the various basic variables. sense and part in a de- stabilising or loading sense, different values of 'Y fa should be used for the two components; 'Y fG .;;; 1 when the load is stabilising the structure and 'Y fG ? The partial coefficients for the new code were determined for use with checking equations of the form (11.42) where fy is the yield stress of the steel, G1 is structural self weight, G2 is superimposed permanent load, Q is traffic loading, 'Ym l is a partial coefficient on yield stress which applies throughout the code, 'Ym 2 is a partial coefficient on the computed resistance which varies with type of component, and 'YfGl' 'YfG2 and 'YfQ are partial coefficients on loads. As mentioned in section 1.3.2, methods of reliability analysis are classified on the basis of the types of calculations performed and of the approximations made. A typical example of a series system is a statically determinate structure as shown in figure 7.4. Obviously, failure in any member of such a structure will result in failure of the total system. " " I I I I "\EICT(g) IIlR ; 78.651 EICT(g)IIlR.optl...l \ "" " ...... Typical examples of gross errors are mistakes in design calculations, use of the wrong size of reinforcing bars or grade of steel, misinterpretations of geotechnical survey data, subjecting the structure to a class of loading for which it was not intended, etc. In equation (11.5) the resistance function r and the load effect function s are shown as uncoupled; and because they share no common variables the two terms are also statistically independent. Their Generation and Propagation on the Ocean Surface. Instead, it is appropriate to use the distribution of the extreme value of the load in the reference period for which the reliability is required; or, where there are two or more time-varying loads acting on a structure together, the distribution of the extreme combined load or load effect. Further assume that c and 2 are realizations of random variables with E[L] 4 m, E[C] 2 m, u L U c 0.25 m and Cov[L, C] 1/32 m 2 • The covariance matrix is then = = - = 16 1 C [1 0.5 = = = 0;5] so that the results from example 6.4 can be used directly. It is therefore to be expected that strength parameters which are affected by friction, (e.g. the shear strength of cohesionless soils, cables, etc.) will tend to be log-normally distributed, since spatial variations in the coefficient of friction J.1 will give rise to expressions of the form (3.44) The Weibull distribution: This distribution is used quite frequently to model the distribution of the strength of a structural component whose strength is governed by size of its largest defect. The corresponding density function fy is illustrated in figure 3.13 and can be compared with the density function of the arbitrary-point-in-time distribution f x ' Since, for a continuous loading process, the largest extreme load that occurs during any reasonably long reference period T corresponds to the largest of a finite number of peak loads, it can be seen from sections 3.2 and 3.3 that the probability distribution of the largest extreme is likely to be very closely approximated by one of the asymptotic extreme-value distributions. The latter are discussed in chapters 7 and 8. (t)fx'..:al University of Denmark, Lyngby, February, 1979. Example 3.3. Consider the mechanical properties of a single nominal size of continuously· cast hot·rolled reinforcing steel. W.: A Survey of Floor Loads in Office Buildings. (3.51) 61 3.6 ESTIMATION OF DISTRIBUTION PARAMETERS (3.52) The equivalent sample moments are ml 1 ='j1L: xi n (3.53) i=l 1 m 2 ='j1 n L; xf (3.54) i=l Hence by equating terms, the estimators p. However, when dealing with single time-varying loads and so-called first passage problems (Le. when failure occurs if and only if the load exceeds some threshold value), the form of the arbitrary-point-in-time distribution is not of immediate relevance. Other distributions: A number of other common distributions exist which may on occasions be useful for modelling the uncertainty in resistance variables - for example, the rectangular, beta, gamma and t-distributions. Let r = p (see chapter 12) up - us . In the normalized coordinate system the failure surface is then (compare with example 5.5) (6.39) The reliability index {3 can now be calculated by an iterative technique analogous with the method used in example 5.5. The only difference is that the new values of J.1'p and up must be calculated after each step in the iteration in the following way J.1~ = x; where x; -1 (6.40) (6.41) = Fpl (({3a 3)). + i1 32816627 m 67108864 - J1 ~ = = 57 + 67108864 = 57.48901 The first random number is the fractional part of aio/m = 0.48901 = r 1 and the new seed for generating the second number is i1 = 32816627. The weighting factors Wi should be selected to represent the previous frequency of usage of each structural component included in the calibration and should '\' Wi = 1.0. be such that ~ Use of the weighted average failure probability rather than, say, the weighted average reliability index means that the target failure probability Pft tends to be governed by the less reliable components in existing codes. . A.: The Forces Exerted by Surface Waves on Piles. , I~; I (2.38) (y). Structural relialxillty theory and its applications. Pf can be calculated from Pf = ~ (7.2) fR,s (r, s) drds Wf where fR,s is the joint probability density function and Wf the failure region {(r, s)lr-s';;;; O}. P. Therefore MR -Ms (5.12) 1 (a R+ a~)2 Let the safety margin M be linear in the basic variables Xl ' ... Let us now re-examine equation (11.2). In (5.21) the so-called mean point (!ll' ... [4.8] Freudenthal, A. It was not necessary to construct new uncorrelated variables in the way presented on page 99. room used for storage of heavy equipment in office premises - Omission of a load or load combination · .. This is a reasonable assumption, as least as far as independence of G and the other variables is concerned, since it may be assumed, for example, that the probability of having an incorrect size of reinforcing bars is unrelated to the yield stress of the bars or to the loads that are subsequently applied to the structure. [10.3] Madsen, H. This definition of the reliability index fJ was used by Cornell [5.1] as early as 1969. 1973. Similarly, small changes in rQ or rm may sometimes have no effect on either the dimensions or the safety of some structural members. , Xn are independent random variables. The subject has benefitted from a large degree of international co-operation which has been stimulated by various bodies - in particular, the Joint Committee on Structural Safety under the chairmanship of J. , x n) to obtain the sequence x~ , x~, ... APPLICATIONS TO STRUCTURAL CODES 11.5.2 Results of calibration Figure 11.5 shows the scatter in the computed failure probabilities for the major structural components designed to the limits of BS 153 which were included in the calibration calculations. There are, however, a number of special cases. [12.32] Rice, S. (3.22) 0 DO. et al.: Measurements of Wind-wave Growth and Swell Decay during the Joint North Sea Wave Project (JONSWAP). The most basic is the so-called arbitrary-point-in-time or first-order dis- tribution of X. For the special case e = 0, the distribution simplifies to the so-called 2-parameter Weibull distribution Fy (y)=l-e - (~)~ -10 1 -100{> 1 -10~ 1 -10-4 normal 1 _10-;\ 1 -10-2 1 _10-1 0.5 10-1 10~2 1O~1 10-4 10~ 100{> 0.5 (3.25) k / / / / / III rit' (/ / / -v'/ w-- /'" / --\:- - ./' ,1-'\ ,/ -- ,....I ,/ ----t- '_ type II maxima "-- r- type I maxima PO' Llog-norm I ',' I /11 ' I y I 0.5 1.0 1.5 2.0 2.5 3.0 3.5 Figure 3.4. Cumulative distributions of different distribution functions (My 4.0 4.5 = 1, 0y = 0.2). et al.: Konstruktioners sikkerhed (in Danish). This probability 163 10.2 THE LOAD COMBINATION PROBLEM is >L P(max X(t) = > n+ P(X(O) where P(X(O) t E [0; TJ) = P(one or more upcrossings of ~ > n is the probability that the process I X(0) U+ vxCO'T and for most practical reliability problems vx(O'T such cases vx(O' T is a good approximation of P(max X(t) P(max (X(t) >L t E >L ~ 1. The specification, involving documents and drawings, will typically contain information of the type - the column shall contain 16 40 mm diameter reinforcing bars - the nominal thickness of the slab shall be 200 mm the structural steel shall have a nominal yield stress of 350 N/mm2 244 13. 1966. t-+~ Failure rates and hazard functions: Also of interest is a knowledge of how the rate of failure changes with time for any particular form of reliability function. This is illustrated in figure 10.1, where realisations PI (t) and P2(t) of two loading processes {PI (t), 0.;;; t.;;; T} and {P2 (t), 0.;;; t.;;; T} are shown together with the sum PI (t) + P2(t). [3.4] Bannister, J. The effect of marine growth is relatively unimportant. J., 1965. 4.3.6 Monte-Carlo Methods Let us assume for simplicity that the basic variables Xi in equation (4.38) are statistically independent with known distribution functions. [3.3] Baker, M. Example 4.2. If Rand S are independent normally distributed variables, Pf may be expressed as Pf = P(M'; 0) (4.21) where M=R-S (4.22) Thus (4.23) 74 4. In case of doubt, the analysis can be started using a range of different trial vectors ZO and searches for the failure surface can be made in a number of predetermined directions. Research Seminar on Safety of Structures, Trondheim, Norway, 1977. 0.: Mathematical Analysis of Random Noise. However, the same procedure can be used for sums of three or more processes. , Zn). The reliability index {3 is then defined as the shortest distance from the origin to the failure surface in the normalized z-coordinate system. C.: Analysis of Simultaneous Wave Force and Water Particle Velocity Measurements. 0 Two practical problems are immediately apparent. dXn (4.41) f(X) .. These methods may also be used for generating log-normally distributed random numbers, by the use of an appropriate transformation. n (6.5) ajaj COV[Xi' Xj] j where equation (6.5) is a generalization of equation (2.86), see also (5.15). In chapter 6 further applications of level 2 methods with special emphasis on non-normally distributed basic variables and correlated basic variables are treated. The structure was subjected to a full structural analysis to obtain natural frequencies, mode shapes and influence coefficients for forces and moments in the various members. be the event [bars are of diameter j] J C be the event [bars are from a single cast of steel] Then in general the density functions f x ' f x / Ai , f x /Bi , f x /Ajn Bj' fX/Ai n Bi nC etc. Naval Architects, Spring Meeting, 1977, Paper No.4. [12.36] Stuugroep Problematiek von Offshore Constructies (Stu POC):Probabilistic Relia- bility Analysis for Offshore Structures. , xn I 0) of a sample with n elements xl' x 2 ' ..• 1 2'··' n .. DIALOG 77. Kirsten Aakj 0 and where fX2 is defined by (2.64). See figure 3.11. [3.2] Baker, M. 11.4.1 Relationship of partial coefficients to level 2 design point It was shown in chapter 5 that for the reliability analysis of a particular structure, the level 2 method involves the mapping of the set of n basic random variables X to a set of independent standard normal variables Z. 1, ... For a discussion of the general theory of random vibrations and spectral analysis see, for example, [B.2) or [B.4). Y., 1973. Reprinted in Wax, N. e.g. see [11.6]. Construction Industry Research and Information Association. National Laboratory of Civil Engineering, Lisbon, Portugal. The reliability index (l may now be defined as the ratio J.lM/oM or the number of standard deviations by which J.lM exceeds zero. 52 3. Those gross errors which cause structural collapse or unserviceability can also be classified according to the type of failure which occurs, as shown in table 13.2. The word failure, here, should be interpreted in the general sense of failure to comply with some performance requirement and not just collapse (i.e. it includes unserviceability). 0 The probability that a system or component, which has already survived for a period of time t, will fail in the next small interval of time dt is thus simply h(t)dt. The sample data shown in figure 3.10 may be considered to be a »good» straight-line plot. A number of desirable properties which characterize »good» estimators are unbiasedness, efficiency and consistency. 974-985. The density function given by equation (2.45) is 1 1 x - JJ. grade 43 steel used instead of grade 50 tion - Incorrect fabrication · .. A flow-chart showing the various steps in the calculation procedure is given in figure 11.4. Define set of structural components and weighting factors wi based on frequency of usage, such that .2Wi = 1.0 Obtain data on load and strength parameters -i ~ Design components to limits of BS 153 Devise suitable probabilistic models for loads and strengths ~ r--1 Determine failure probabilities Pf (153); + .Ewi Pf (153). 95. The planning and coordination of these various tasks is the subject of quality assurance. If the probability density function fx of a random variable X is interpreted as the limiting case of a histogram or relative frequency diagram as the sample size tends to infinity, the probability P given by P = P(x 1 < X";;; ,X 2 x2) =\ , Xl fx(x)dx (3.28) 45 3.4 MODELLING OF RESISTANCE VARIABLES· MODEL SELECTION Hi 0.03 10 0.02 90 , I 110 130 150 170 190 210 Figure 3.5 Number 18 14 t t Mean value Standard deviation No. of readings 153.1 mm 12.7 mm 272 nominal value mean measured value I I I I 10 I I I 6 2 t(mm) 110 120 130 140 150 160 Figure 3.6. Histogram of slab thickness measurements. , (5.20) Xn) By expanding this relationship in a Taylor series about (Xl' ... Struc. Combined cumulative frequency diagram for 12 mm mild steel plates from three mills. These data were not, in fact, obtained by measurements in real structures but were randomly sampled from a logarithmic normal distri~ution with a mean!1T = 150 mm and a coefficient of variation VT = 0.15 (see ap- pendix A). A more direct approach to the choice of target failure probabilities has been recommended by the Nordic Committee on Building Regulations (NKB) [11.10]. For the general case, closed-form solutions do not exist for the integrals in equations (4.17) and (4.18). The random variable which is of importance is the magnitude of the largest extreme load that occurs during the reference period T for which the reliability is to be determined. Example 11.3. The encastre steel beam shown in figure 11.3 is to be designed against plastic collapse to resist a uniformly distributed superimposed load Q and a permanent load G. Furthermore, because n Eat = 1 i=l it is .always possible to choose a conservative set of sensitivity factors for use with equation 11.4. METHODS FOR THE EVALUATION OF PARTIAL COEFFICIENTS 191 (11.26). Obtaining the solution to equations (11.38) and (11.39) is a problem of constrained minimisation for which a number of standard techniques and computer programs are available. It presents an accessible and unified account of the theory and techniques for the analysis of the reliability of engineering structures using probability theory.This new edition has been updated to cover new developments and applications and a new chapter is included which covers structural optimization in the context of reliability analysis. Equation (3.6) ~ay be intkrpreted ihan rence of the event (X I analogous manner. As far as the imposed loads are concerned, the probability of failure of the bridge by a simple plastic collapse mechanism depends only on the weight of the heaviest vehicle (assuming that only one vehicle can be on the bridge at anyone time). Statistics, Vol. In many cases it may not be possible or convenient to express equation (11.4) in explicit form in which case the design process will involve a number of trial-and-error calculations to find the minimum value of d' that satisfies the inequality (11.3). 1st International Conference on the Behaviour of Offshore Structures, Trondheim, 1976. , k taking due account of any constraints (e.g. 0 (3.60) < & < = for the parameter a of a normal distri- bution). Journal of Meteorology, Vol. Show that tl t2 A ~I< Q/2 "v Figure 6.2 Q/2 .,.v Q = 10 m B~ f A' 98 6. A formal method for doing this is discussed in section 11.4. 11.3.4 Treatment of material properties We shall restrict our attention to the strength properties of structural materials, denoted E. Safety of Structures under Dynamic Loads (ed. See figure 4.3. Hence P = (- J.lM) = (- (l) C oM (4.27) For the more general case where Rand S are jointly normally distributed with correlation coefficient p, equation (4.27) stilI holds but oM is given by OM = (o~ + o~ + 1 2poRoS)2 (4.28) Example 4.3. If Rand S are both log-normally distributed, Pc may be expressed as PC =P(M''';;;l) (4.29) where M' = R/S (4.30) 75 4.3.3 PROBLEMS REDUCING TO THE FUNDAMENTAL CASE Taking logarithms to the base e and putting M = QnM' gives M = QnR - QnS = A - B (4.31) where A = QnR and B = QnS. Konstr. John Wiley and Sons, 1977. Let us assume, however, that there are good a priori reasons for assuming that a particular basic random variable X is governed by a particular type of probability distribution. In this case, it is necessary to use numerical differentiation, but this rarely causes difficulties. p.(t) ~ i=l (B.13) J11 where n is the number of lumped masses and P.(t) I Using complex number representation (L x. This will now be generalized to random vectors, where the random vector Y = Yn) is a function f = (f l , ... See, for example, figure 3.6. Geometrical imperfections: The strength of many structural members, for example most plates, columns and shell structures, depends not only on cross-sectional and overall dimensions but also on the magnitude of relevant geometrical imperfections I. These can be considered as a continuous-state/discrete-time stochastic process. [3.14] Mitchell, G. 79 4.3.6 MONTE-CARLO METHODS These difficulties can be overcome in practice by using the level 2 methods described in chapters 5 and 6. Show that the variable representing the largest extreme with distribution function (Fy(y»n has the same coefficient of variation. There are some notable exceptions to both these generalisations. 100, [10.9] Wen, Yi·Kwei: Statistical Combination of Extreme Loads. 60 3. All the other variables are assumed to be deterministic. Cambridge University Press, 1969. Calculation of the reliability index has been shown in chapter 5 by a number of examples, but only uncorrelated basic variables have been treated. Y., 1975. [12.27] Moan, T. 11.4.2 Approximate direct method for the evaluation of partial coefficients The difficulty with the approach suggested. and Handscombe, D. Symposium, London, April 1980. J.: Application of Bayesian Decision Theory. What is the form of the hazard function? LEVEL 2 METHODS In a level 3 method knowledge of the joint probability density function fX- is required, but in the level 2 method presented in this chapter only the expectations i = 1, ... An alternative approach in determining sensitivity is to examine the effects of moderate changes in the distribution parameters. Exercise 3.5. Derive the maximum-likelihood estimators p. If it is assumed that certain components, such as welded joints, contain a large number of small defects and that the severity of these defects is distributed in an appropriate manner, the distribution of the component strength approaches the Weibull distribution. In the former, he or they are just unfortunate, unless it can be shown that currently accepted practice has been extrapolated to an unreasonable extent. Then a new set of variables Z = (Zl' ...• Zn) is obtained by normalizing the original set of basic variables X in the following way Z. 4.3 STRUCTURAL RELIABILITY ANALYSIS 4.3.1 General Attention will now be focused on the particular features of structural reliability analysis, but first, some of the principal differences between electronic/mechanical systems and structural systems will be examined. 113 Chapter 7 RELIABILITY OF STRUCTURAL SYSTEMS 7.1 INTRODUCTION In the preceding chapters the concept of reliability has mainly been concerned with single structural elements such as a beam or a column. [A.3] Tocher, K. A suitable general method for the evaluation of such a set of partial coefficients is now presented. 121-147. above was seen to lie in the evaluation of suitable sensitivity factors Ii. Experience shows that over fairly large ranges of design parameters the individual factors a i often do not change dramatically. In other words, the variable mensional basic variable space. and Muller, M. For low failure probabilities and/or small n, the estimate of Pf given by kIn may be subject to considerable uncertainty. Mark: Random Vibration in Mechanical Systems. Equations (11.6) and (11.16) are the most general form of checking equations that are envis- 188 11. It should be noted, however, that the best unbiased estimator of a 2 is not &2 butS 2 =(n/(n-1))u2. Characteristic values of actions and material properties based on a prescribed probability p of not being exceeded were considered to be more rational than arbitrary selected values. 82 5. , Xn can be expressed (2.84) Exercise 2.12. [13.13] Moan, T. For many structures it is possible to re-write this as X R r.(E, DR' c) - Xs s (Q, G, Ds' c) >0 where r represents a resistance function and R s represents a load effect or action effect function and S = s (. ax Exercise 9.3. Consider an ergodic narrow-banded Gaussian process {X(t)}. Substitution yields, E[Qn8] = 4.3923. This is still continuing. Div., ASCE, EM3, June 1971. J Example 11.1. If Xj is a normally distributed loading variable, then (11.27) 190 11. The main intention is to give the necessary background for understanding the ideas behind an approximate method for dealing with load combinations. LEVEL 2 METHODS Exercise 5.1. Prove equations (5.22) and (5.23) and derive the special form of (5.23) when the basic variables are uncorrelated. is the sample mean. This is best illustrated by means of an example. Take a failure function of the form f(G, R, K, S) o and let the safety margin M be M = GR-KS = (13.1) where R is a continuously distributed random strength variable, N(}JR' a R) S is a continuously distributed random load effect, N(}JS' as) K is a discrete-valued model uncertainty, and G is a gross error which modifies the strength parameter R, and assume that the quantities R, S, K and G are statistically independent. However, the probability of failure by fatigue will also depend on (a) the weights of the other vehicles and (b) whether the individual vehicles induce any appreciable dynamic response. Consider again the two jointly distributed discrete random variables Xl and X 2 from example 2.13. The main core of the book is devoted to the so-called level 2 methods of analysis which have provided the key to fast computational procedures for structural reliability calculations. The prevention of failures which arise from lack of knowledge within the profession as a whole is clearly impossible and occasional failures of this type will continue to occur. Graphical procedures: For most simple probability distributions, it is possible to plot the cumulative distribution function Fx for different values of the variable x as a straight line, simply by pre-selecting an appropriate plotting scale or type of probability paper. e, p > 0, k > e? , (3O:n)' where (s2e (5.35)) i = 1, 2, ... Sharp practice ABCDEFGH Figure 13.1. Analysis of underlying causes of 120 structural failures in buildings, from [13.8 \. R. Care must also be taken when there is appreciable statistical uncertainty in the parameters of the probability distributions of the basic variables because of lack of data. Let the random variable X have a 2-parameter Weibull distribution with parameters (3 and k and distribution function FX(x) = 1 - exp(-- (x/k)li) Then, z = £n(- £n(1 - x ~ 0, (3 > 1, k> Fx (x») is a linear function of y 0 (3.61) = £nx, since £n(- £n(1 - Fx(x») = {3£nx - {3£nk (3.62) The variables x and y therefore plot as a straight line on natural scales. , Xn is not the whole R n space, one can apply one-to-one transformations to the relevant variables. [12.15) Fjeld, S., Andersen, T., Myklatun, B.:Risk Analysis of Offshore Production and Drilling Platforms. 249 Appendix A RANDOM NUMBER GENERATORS 1. The reliability index defined by equation (5.9) is thus not invariant with regard to the choice of failure function. Determine Pft = • 1 I i Choose approximate values of partial coefficients r- - - - - - - - - - - -- - Modify partial coefficients ~~ r NO Does P = Pft and is S a minimum? The logarithmic normal distribution: The logarithmic normal (or log-normal) distribution is frequently used for modelling resistance variables and has the theoretical advantage of precluding negative values. and N. 1 are the mean and the standard deviation of Xi' By the linear mapping (6.15) the failure surface in the x-coordinate system is mapped into a failure surface in the z-coordinate system. It is also of interest that if Y is type II maxima distributed, then Z = 2nY is type I maxima distributed. 3rd International Conference on the Application of Statistics and Probability in Soil and Structural Engineering. Mech. Struct. This method is very suitable for use in connection with the level 2 methods presented in chapters 5 and 6. Whereas the former is simply a record of observations, the latter is intended for predicting the occurrence of future events - e.g. a thickness less than 100 mm. Because of the inherently probabilistic nature of most structural safety problems, it is clear that safety checking procedures which are couched in deterministic terms will have some degree of arbitrariness. An obvious limitation of this work is that it was restricted to a study of structural components rather than structural systems. 1981. Prentice-Hall, Englewood Cliffs, N. 1970. With M = R - S one gets (5.10) and ait = a R+ a~ (5.11) according to equation (2 .86). For example, the self-weight of a reinforced concrete beam and hence the mid-span bending moment S will be weakly correlated with the beam's moment-carrying capacity R, as both are functions of beam depth. Equation (3.33) represents what is known as a mixed distribution model. 29, 1958. Pij is equal to the correlation coefficient PXiX j defined in equation (2.80) . j Cx (defined (6.9) Var[X n] x Clearly, no correlation between any pair of random variables will exist if this matrix C is a diagonal matrix. The following recommendations for level 1 codes are based on the work of the International Joint Committee on Structural Safety [11.7], [11.8], and are likely to form the basis of a new international standard to replace ISO 2394: General principles for the verification of the safety of structures. Extreme caution should therefore be exercised if the type of probability distribution is to be chosen on the basis of sample data alone. & C. The term action is now often used as a more general description to include both loads and imposed deformations. R.: Statistical Prediction Analysis. BIBLIOGRAPHY [12.1] American Petroleum Institute: Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms. The reason for this is discussed in example 3.4 below. A level 1 code is therefore a conventional deterministic code in which the nominal strengths of the structural members designed to that code are governed by a number of partial coefficients or by equivalent means. Most loads differ from other basic variables in that they vary significantly with time and are generally not amenable to effective control. Example 3.2. For i = 1 equation (3.4) simplifies to: (3.6) This is the distribution function for the minimum value in a sample size n. The combination problem can then be formulated in the following way. [2.4) Ditlevsen, 0.: Uncertainty Modeling. , ns (11.35) Hence, for the loading variables R1 and Sl' a R 1 = as 1 = 1 giving lIR 1 = - 0.8 and lIS 1 = 0.7. " , , This approach is viable only if the designer has prior knowledge of the relative importance (sensitivity ranking) of the various variables. 106, No. St. 12, December 1980. RELIABILITY THEORY AND QUALITY ASSURANCE Blockley, D. Cl: 1 = ± Cl: 2 ~ ..• = ± Cl: n = ± 1 is such a set, when the sign of the factor is taken as positive for loading variables and negative for resisting variables; although in most practical cases this would be too conservative. This does not mean that reliability theory cannot be used under these conditions - it means that the models have to be amended. (11.30) ,1 Cl: S,I· where Cl: R . BAKER, B.Sc. (Eng) Department of Civil Engineering Imperial College of Science and Technology London, England Library of Congr~S5 ('ataloging In Publication DatH Thoft-Christensen, Falle. The next step is then to normalize the uncorrelated variables and thereby obtain a set of normalized and uncorrelated variables Z = (Z 1, ... L.: Systems of Frequency Curves. be the distribution 1 function for the strength Ri of element i, then =1 - (1- FRI (r 1))(1 - FR2 (r 2)) ..• (1 - FRn (r n)) =1 - 11 (1- FR/r n 1=1 i)) (7.3) where it is assumed that the strengths of the elements are independent. will all be different; not only their parameters but also their shapes. Therefore, one must not interpret figure 7.5 as one in which all elements have the same load, although a load S is shown applied to each end of the series system. and fx X are the joint density functions for Xl' Xl and X 2 , X 2 , respectively. Report 63, Construction Industry Research and Information Association, London, England, 1977. , Y n) as shown in the last section. (x) = 1 J r~ J_ (10.15) f x . Offshore Tech. EXTENDED LEVEL 2 METHODS where IlX. [5.2] Ditlevsen, 0.: Structural Reliability and the Invariance Problem. The next number in the pseudo-random series is related to the previous number by the relationship (A.3) rn+ 1 = arn (modulo m) where a and m are integer constants and are relatively prime. severe impact or explosion · .. , Xn), (2.71) where i = 1, 2, ... Eng., Chalmers University of Technology. These problems will not be discussed in detail since they are different from the systems problems that are encountered in structural engineering. This book emphasises concepts and applications, built up from basic principles and avoids undue mathematical rigour. It will now be assumed that the load P is Gumbel distributed with the distribution function (see (3.10)) Fp(p) = exp(- exp(-- a (p - u))) and the density function fp(p) exp[- exp(- a(p - u)) - a(p - u)la = The two parameters a and u can be calculated from the following expressions for the mean J.1p and the standard deviation up (see (3.11) and (3.12)) J.1p = u u p + 0.57722/a 1 a 7f =-.- V6 when J.1p and a p are known. Let the strength Re of each tensile bar be a random variable with the density function fRe shown in the same figure. But if Rand S are lognormally distributed, A and B are normally distributed, so that M is also normally distributed. H.-S. -p.)M .. RELIABILITY THEORY AND QUALITY ASSURANCE Type of failure Gross error Type A: Errors affecting: Failure in a mode of behaviour against which the structure was designed Type B: Failure in a mode of be- - load-carrying capacity ability to remain serviceable applied load(s) - Errors that relate to the fundamental understanding of structural behaviour, arising from: haviour against which profession's ignorance the structure was not - designed - engineers' oversight engineers' ignorance Table 13.2. Classification of gross errors according to type of failure. The selection of probabilistic models for basic random variables can be divided into two parts the choice of suitable probability distributions with which to characterize the variability in each case and the choice of appropriate values for the parameters of those distributions. This is due to the fact that in such cases the distribution of the maximum value of the loading process in a given reference peri ode can be derived from the arbitrary-point-in-time distribution (see figure 3.13 on page 57). and ax' 1. & W. [12.25] Malhotra, A. The »time to failure» may, however, be recorded in terms of number of cycles of operation, number of miles driven, etc., rather than in units of actual time. y P m By consideration of the mean values and coefficients of variation of the variables and the nature of the limit state function it may be assumed that Thus, =aSaS,1 =0.7 X 1.0 =0.7 G: E =aR aR 1 =- 0.8 X 1.0 =- 0.8 G: xm =aR aR ,2 =- 0.8 X (.;2 -1) =G:G =a SaS,2 =0.7 X (.;2 -1) =0.290 G: Q y , 0.331 Variable IlX ax Vx xsp Q kN/m 40.1 6.015 15% 50.0 G kN/m 30.0 1.5 5% 30.0 Ey N/mm2 293.6 23.49 8% 255.0 1.0 0.06 6% 1.0 Xm Table 11.1 J I I I I I I I I I I I I IG,Q r-----Figure 11.3 --0-- B --/={ V 193 11.4. METHODS FOR THE EVALUATION OF PARTIAL COEFFICIENTS and the design values x· are given by x· m = J.i x m + ax m ~aX m = 0.921 These values and the partial coefficients found from equations (11.25) and (11.26) are listed in table 11.2. Variable xsp x· r Q kN/m 50.0 56.9 1.13 G kN/m 30.0 31.7 1.06 Ey N/mm2 255.0 218.4 1.17 1.0 Xm 0.92 1.09 Table 11.2 By application of virtual work, the required plastic modulus zp may now be determined from Q2 _ 4(~) 10~ (rGgsp + rQqsp) 4r zp r-- Ey (11.36) Xm Substituting the appropriate values from table 11.2 gives zp = 6.89 X lOs Finally, it is of interest to use the level 2 method to determine the reliability of this structure when the plastic modulus has the value found by the above method. 92, No. ST1, Feb. EXTENDED LEVEL 2 METHODS t fp(p), np(p) 05 1 0.4 0.3 0.2 0.1 O.O+-~~~~---4~---+----~----1-----+---~~==~-----+--~p 2 3 4 5 6 7 8 9 Figure 6.6 The reliability index is now (3 = 3.32 which is a small increase from example 5.5, where (3 = 3.29. , k Example 3.12. 3.5 MODELLING OF LOAD VARIABLES· MODEL SELECTION 55 It is often helpful to classify loads and other actions in accordance with the following three attributes [3.9]. 50 3. Up to the present date (1982) few national loading committees have attempted to rationalise their specified loads along these lines, but progress is being made in this direction. C may be diagonalised by error minimization as in the drag term linearization. The exponential density function is given by (4.10) and where X is a constant. Journal, Vol. The design process generally involves iterative or trial-and-error calculations to find a set of dimensions dRd which in conjunction with the design values of the load and strength variables satisfies the checking equation. 4.3.4 Treatment of a Single Time-Varying Load The situation discussed in section 4.3.2 was that of an uncertain load effect S applied once to a structure of uncertain resistance R. If Qc is the total length of reinforcement produced from a single cast of steel then the average yield stress for the cast can be defined as = u yd 1 ~Qc =Q u yd dQ c 0 (3.31) 47 3.4 MODELLING OF RESISTANCE VARIABLES - MODEL SELECTION 500 480 460 440 Z(m) 420+---~-----+-----r----1-----r---~-----+--~_ o 500 1000 1500 2000 2500 3000 3500 Figure 3_7 Variations in dynamic yield stress along a 20 mm diameter hot-rolled reinforcing bar. For this reason it is necessary to introduce a set of reduction factors lji OJ (lji OJ .;; 1) to be applied to the time-varying loads Qj to take account of the reduced probability of the design values of the loads being exceeded simultaneously. M., Garretts, J. &2 =- "(x. It is therefore necessary to find the inverse function F-1 , giving x = Fit (r) (A.6) This is valid for all forms of distribution function, but two classes of variable exist which require different treatment. and Wyatt, T.: Methods of Reliability Analysis for Jacket Platforms. (Jyd (i) is the yield stress of the ith bar and n is the number of bars rolled from the If we are interested in the statistical distribution of the yield stress of reinforcing bars sup- plied to a construction site, account must also be taken of the variations in ayd that occur from cast to cast. It is important to remember that no information regarding the probability of failure can be ob- tained when the distributions of the basic variables are unknown. , X n). It will also be used here, to cover all aspects and stages of a structural development, but it should be emphasised that it is applicable to all structural projects, not just buildings. Lack of ability to communicate E. 1 = 10.25 (kNm)2 4 (5.18) Therefore 3.12 (5.19) Note that in the presentation above, a safety margin linear in the basic variables has been assumed. This is larger than the originally selected value of 4.0 showing that the approximate method of determining partial coefficients is safe, at least for the structure and set of variables examined. By analysing a realization of this process it is concluded that the expected rates of positive crossings of the barriers ~ = 0, 5, and 10 are 10-2 ,10-3 , and 10-5 , respectively. The procedure is as follows: 1) Select a trial vector ZO , preferably in the region of z* . The preceding example is somewhat simplistic and little attention should be paid to the actual numbers used. must be calculated for those variables I I where such a transformation has been used. For example, although snow loading may dominate the load effect in the roof beams of a multi-storey building, the same loads have only a small influence on the total load effects in the ground floor columns. Uncorrelated normalized variables Zl and Z2 can be calculated from equations (6.16) and (6.17). The relative importance of Cd compared with Cm is also not unexpected . , f.in) and retaining only the linear terms one gets (5.21) where i3f/i3X j is evaluated at (f.il' ... It can be shown that - 1 .;;; PX l x2 .;;; 1. & Cornell, C. x,y x(t) o t' T Figure 3.13. Paper 3046, Houston, 1978. This is best explained by means of an example. All that can be said is that if a variable X does in fact have a known probability density function fx' and if it is sampled at random an infinite number of times, the proportion in the range 1xl' x2 [will be P. Y., 1965. When data from two or more sources are present in a single sample, the overall shape of the cumulative frequency distribution is likely to depend as much, if not more, on the relative number of observations from each source than on the actual, but unknown, probability distribution of each SUb-population. BIBLIOGRAPHY [2.1) Ang, A. RELIABILITY THEORY AND QUALITY ASSURANCE material properties. Petroleum Transactions, AIME, Vol. 0 where f Xl •x 2 • • • . But E[Qn8 J = !1 Qn (0 = Qn(m e) from equation (4.35). Thoft' Christensen), Aalborg University Centre, Aalborg, Denmark, 1980, pp. Taking a sample size of n (e.g. n years records and n values of the maximum mean-hourly wind speed) let the cumulative distribution function of the ith smallest value Xi in the sample be F x.n and its corresponding density function be fx~ 1 . It is clear that (4 1. This is discussed in section 11.4. It should be noted that in practice the quantities Rand S may often be correlated beacuse of common parameters. and Wyatt, T. Care must be taken, however, to ensure that the data are effectively homogeneous and do not include, for example, systematic changes with time. English Edition, 1980. The log-normal distribution arises naturally as a limiting distribution when the random resistance X is the product of a number of independent identically distributed component variables, i.e. n X = Z1 Z2 ... Assume, in addition, that a particular random sample (Xl' x 2 ' ••• , xn) of the random variable X has been obtained. Since each material and mechanical property is different, each requires individual attention. There is a need for all structural engineers to develop an understanding of structural reliability theory and for this to be applied in design and construction, either indirectly through codes or by direct application in the case of special structures having large failure consequences, the aim in both cases being to achieve economy together with an appropriate degree of safety. on Probabilistic Mech. L.: Waves and Wave Spectra and Design Estimates. For example, with time-varying loads, the analyst is interested in the likely value of the greatest load during the life of the structure. Let the structure be loaded by a single tensile force S = 1.1 kN. In calculating this value, stiffened compression flanges and unwelded plate panels were excluded, the former because they had not been shown to behave satisfactorily in service and the latter because the data on model uncertainty were considered inadequate. For example, the random variable (defined by equation (A.13) cannot lie outside the the interval [- 6,6]. It should be noted that the modelling did not allow for the possibility of gross errors in design or construction and for this reason the probabilities should be interpreted as a measure of relative safety and not as failure frequencies. It is desired to evaluate the partial coefficients 'Y Q' 'Y G' 'YE and 'Yx for a reliability index ~ = 4, and to determine the required plastic modulus z . It should be noted that because of the systematic decrease in reinforcing bar yield stress with increasing diameter, equation (3.34) gives rise to a density function fx I A. The main exception of course is the selfweight of permanent structural and non-structural components. However, when the safety margin M is linear in the basic variables Xi' i = 1, ... , n is considered a realization of a random variable Xi' i = 1, ... [12.19) Houmb, O. In such cases, the partial coefficients should be used only with the precise form of the design equations (failure functions) for which they were derived. Comite Euro-International du Beton. and Ramachandran, K.: Reliability Analysis as a Tool in the Design of Fixed Offshore Platforms. A procedure is therefore required for the determination of a limited number of partial coefficients or additive safety elements « n, where n is the number of basic variables) which will be applicable over a range of different failure modes and for a range of different structural types covered by a code of practice. By (10.7) the problem of calculating the distribution function F:=: for the maximum value of {XC t)} = {Xl (t) + X 2 (t)} is reduced to that of determining the rate of up- crossings (the expected number of positive crossings) vxCO = E[N~ enJ for {X(t)}. In a study of 120 building failures, it was found that over 60% were due to lack of experience by the personnel concerned and that in about 50% of all cases the major cause of failure was a lack of appreciation of the relevant design concepts - for example, ignorance of the need to design against lateral torsional buckling in unsupported compression flanges. Nevertheless, each of the features mentioned above is of relevance to future code development. For example, in a random sample of 10 independent observations drawn from a normal distribution, there is a probability of approximately 1 :1000 that by chance all observations will be greater than the mean. 2) Determine af, i = 1, 2, ... For a wide range of structural members, the following empirically-based values can be shown to be satisfactory (11.33) &R,I. Stages 1) and 2) of the building process are (or should be) intimately related. The book does not try to cover all aspects of structural safety and no attempt is made, for example, to discuss structural failures except in general statistical terms. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES Finally, it should be noted that some »loads» act in a resisting capacity for some failure modes for example, a proportion of the self-weight of the structure in most over-turning problems. ANDRÉ T. CIRIA Technical Note 44, September 1972. Thus, equation (B.ll) may be rewritten for mode j as (M = M jj , C = Cjj , K = K jj) (B.12) or Mx.J + Cx.J + Kx.J n = ~'l1 .. [9.2) Krenk, S.: First-Passage Times and Extremes of Stochastic Processes. These problems tend not to arise in simple structural problems, but for complex structures more care has to be taken. Equation (11.1) may therefore

be re-written as 181 11.3. RECOMMENDED SAFETY FORMATS FOR LEVEL 1 CODES (11.2) For the purposes of a level 1 code, the equivalent deterministic criterion for safety checking (i.e. checking the sufficiency of a structure or structural member whose design properties are given) is (11.3) where f is the same limit state function as above, involving n quantities xd and m constants c, and qd is the deterministic design value of the random variable Q, etc. [13.2] Blockley, D. This approach gives excellent approximations to the normal distribution for deviates within two or three standard deviations from the mean, but for extreme values the approximation becomes increasingly poor, unless n is large. It is of course unfortunate that a reliability measure can give different values for the same problem. 75-98. It can be seen that there is a fairly strong positive correlation between Uyd for adjacent lengths, as might be expected. In this chapter we consider how these techniques can be used in the development of conventional structural codes. [4.4] Bolotin, V. For instance, errors can be classified according to nature of the error (table 13.1) type of failure associated with the error (table 13.2) consequences of failure arising from the error those responsible for causing the error - those responsible for not detecting the error etc. Fx(x) 0.998 0.995 0.99 0.98 0.95 - 0.9 t--- 0.8 0.7 0.6 0.5 0.4 ['x,.) ~ ABC .r 0.3 0.2 ..j 0.1 ..... As mentioned earlier, timevarying quantities are best modelled as stochastic processes, but discussion of this topic is postponed to chapters 9 and 10. This system can be solved iteratively in the same way as the system (5.39) in example 5.5 was solved. It is then clear that the possibility exists for using any simplified set of design clauses together with a modified set of partial coefficients which on average will achieve the same degree of safety as the more complex set. A discussion of this large subject is beyond the scope of the present text but the reader is referred to [13.9] for further study. [10.8] Turkstra, C. In the following it is assumed that a set of basic variables X = (Xl' ... Any attempt to estimate the parameters (Il, a) of the parent distribution from this particular sample will result in considerable error. In the fundamental case the loading is described by a single random variable S and the strength by a single random variable R (see chapter 4). , n are one-to-one func- (2.72) It can then be shown that (2.73) where i = (xl'· .. This detailed deterministic analysis was necessary to provide input for the simplified models used in the reliability analysis and to interpret the response in terms of stresses, as discussed in section 12.2. The reliability analysis was undertaken to check the safety of the structure under the action of wind, wave and current loading and to assess the sensitivity of the design to the various random variables that affect its behaviour. The probability distribution of X, therefore, tends towards the log-normal, as n increases. See also [11.11]. Many structural engineers are shielded from having to think about such problems, at least when designing simple structures, because of the prescriptive and essentially deterministic nature of most codes of practice. , =-1 (2.89) _ x n), M = C , and where C is the covariance matrix defined by (2.84). (Whether this is true in practice clearly depends on a number of other factors and whether these assumptions are valid). It should be clear that because of the random nature of the variable no sample, however large, is completely representative of the source from which it derives; and indeed, small samples may be markedly unrepresentative. Provided the correct distribution is used, the time element may then be neglected in the reliability calculations. This uncertainty may be represented by modelling L as a lognormal variable with a mean of 4 m and a coefficient of variation VL = 0.15. However, for small sample sizes, the shape of the histogram varies somewhat from sample to sample, as a result of the random nature of the variable. 1977. t. BIBLIOGRAPHY is approximately normally distributed with zero mean and unit standard deviation. [12.11) Chakrabarti, S. This is an undesirable situation. As discussed in chapters 9 and 10, time-varying loads are best modelled as stochastic processes, but this is not a convenient representation for use with the methods of reliability analysis being presented here (chapters 5 and 6). 13.3 INTERACTION OF RELIABILITY THEORY AND QUALITY ASSURANCE 13.3.1 General The phrase »building process» has been used in the preceding sections as a general term to include planning, design, analysis, construction, maintenance and use of a structure. It should be noted that the sensitivity factors a i (discussed in chapters 5 and· 6) are not directly calculated using the above procedure, but may be evaluated from i=1,2, ... When the total permanent load acting on a structure is the sum of many independent components, it can easily be shown that the coefficient of variation of the total load is generally much less than those of its components. Show that n n j=l j=l E[ll fj(X j)] = II E[fj(Xj)l (2.78) when Xl' ... Then Fx is the arbitrary-point-in-tiine distribution of X(t) and is defined by Fx (x) = (3.45) P(X(t') ,,;;; x) where t' is any randomly selected time. Show that (2.86) 2.10 MULTIVARIATE DISTRIBUTIONS The most important joint density function of two continuous random variables Xl and X 2 is the bivariate normal density function given by (2.87) where - and III ,1l2 are the means, uI ' u2 the standard deviations and p the correlation coefficient of Xl and X 2 • 00 .;;; xl .;;; 00, - 00 .;;; X2 .;;; 00, Exercise 2.13. How should spatial variations in this load be taken into account? University of California, Berkeley, 1978. Hence the lack of availability of statistical data on extremely low strengths is not too important, for such cases. A typical load-deflection curve for a brittle element is shown in figure 7.1. If an element maintains its load level after failure it is called perfectly ductile. Finally, the estimates of the distribution parameters are obtained from the slope and position of the best straight line. For example, see figure 3.13 which shows a continuous state/continuous time stochastic process. 44, 1945 (Reprinted with [12.32]). Edward Arnold, 1966. This example indicates that, at least for the set of models and parameters chosen, the possibility of the occurrence of gross errors should not influence the selection of partial coefficients for use in structural design. Example 13.2. (taken from [13.1]). Trans. Class A: The distribution function Fx has an inverse Fil which can be expressed in closed form In this case the random deviate x can be generated simply by obtaining successive values (A.7) Example A.2. Let X be type I extreme (maxima) distributed with distribution function FX(x) = exp[- exp(- ex(x - 'Y»)l (A.8) and parameters 'Y and ex. One approach is to use single point estimates for the parameters - for example, the maximum-likelihood estimates - and to ignore the additional statistical uncertainty that arises when there are too few data. As discussed in section 3.3.3 it is one of the so-called asymptotic extreme value distributions. Structural dimensions: The uncertainties in most structural dimensions D are generally small and for this reason the mean value IlD may be taken as the characteristic value (i.e. d k = IlD)' Tolerance limits are specified in codes for most structural dimensions, and if these are of the form dsp -f.;;;D.;;;dsp +1' (11.8) 184 11. [12.31] Pierson, W. Lack of formal qualifications B. As previously mentioned (for example in chapter 3), the probabilistic models used in reliabilityanalysis are conditional upon the specified standards of quality control and acceptance tests for the materials, and on the standards of inspection for the finished structure. Determine the joint probability mass function px: for the random vector X = (Xl' X 2). This surface is commonly referred to as the failure surface for the limit state under consideration and divides all possible combinations of the variables X which cause failure from all possible combinations which do not. [12.38] Wiegel, R. 2.9 FUNCTIONS OF RANDOM VARIABLES In chapter 2.5 a random variable Y, which is a function f(X) of another random variable X, was treated and it was shown how the density function fy could be determined on the basis of the density function f X, namely by equation (2.38) fy(y) = fx(x) where x = f- I (YI' Y2 , that is ... Use was made of the JONSWAP spectrum to relate extreme wind speed to sea-state. Paper 3152, Houston, 1978. Some rather less general forms of checking equations have also been suggested [11.7]. The reader is warned against a too literal interpretation of some of the simple examples as these were not included to provide insight into particular practical problems. 51 3.4 MODELLING OF RESISTANCE VARIABLES - MODEL SELECTION We now return to the question of selecting a suitable probability distribution to model the uncertainty in the strength variable X. Formally, if Y1 , Y2 , ... It is therefore the pth fractile of the extreme value distribution of the load corresponding to that reference period. The overall process of parameter estimation consists of • initial inspection of the data • application of a suitable estimation procedure • final model verification. In other cases, the analyst may prefer to use some prescribed distribution type to model a basic variable, e.g. a log-normal distribution to model a resistance variable, even though over the limited range of available data some other distribution type may in fact give a better fit. LEVEL 2 METHODS Example 5.2. Consider the fundamental case treated in example 5.1 and assume that Rand S are uncorrelated. As will be discussed in section 11.4, the most consistent standards can be achieved by associating a partial coefficient or some other safety element with each major source of uncertainty (i.e. with each basic variable). It is obvious from these observations that knowledge of the detailed time variation of the two loading variables in the reference period T is required to determine the probability distribution of the sum of the two load variables. Bygningsstatiske Meddelelser, Vol. This topic is discussed in the next two sections. It must be stressed that great care must be taken when using the spproximate method for the evaluation of partial coefficients if the relative magnitudes of the sensitivity factors ex are not known. This was discussed in chapter 10 in the context of reliability analysis. An illustration of this is given in example 6.5 for the 2-dimensional case. Very often one needs to calculate the mean and variance for a random variable Y which is a linear function of a number of random variables Xl ' ... Consider the yield stress of a steel bar. In particular it is assumed that the failure surface described in 5.2 below can be sensibly approximated by a tangent hyperplane at the point on the failure surface closest to the origin, when the surface has been mapped into a standard normal space. 233 12.5 SOME RESULTS FROM THE STUDY OF A JACKET STRUCTURE Variable Distribution Annual extreme 6-hourly wind Type I extreme 26.1 mls ex aO 110 3.3 + 0.900 Yield stress Log-normal 380 N/mm2 18.0 - 0.214 Current speed Normal o mls 0.14 Cd Marine growth at m.w.l. Normal 0.75 0.225 Normal 0.16 m 0.06 + 0.203 + 0.199 + 0.154 Strength model uncertainty Normal 0.067 0.023 - 0.099 Cm Deck load Normal 1.80 0.27 Normal 24000 t 720 + 0.063 + 0.047 Thickness of leg wall Normal 33.5 mm 0.34 - 0.046 Leg diameter Normal 4191 mm 16.8 - 0.013 Damping ratio Normal 0.03 0.01 - 0.001 Others - - - (0.146) Table 12.1. Analysis of jacket structure. Ellis Horwood, Chichester, 1980. In general, it is not possible to relate the reliability index {J defined by (5.9) to the probability of failure Pf given by Pf = \' f:x(x)dx • Wf (5.27) where f:x is the joint probability density function and w f is the failure region defined earlier in this section. Two random variables Xl and X 2 are said to be uncorrelated if PX l X2 = O. Also note that the first step in calculating fxx is to calculate fx 1 X 1 and fx X . Generation of random normal deviates using method due to Box and Muller Box and Muller [A.l] have shown that if r 1 and r 2 are independent random variables from the same rectangular distribution in the interval [0, 1], then Nl and N2 given by 1 Nl =(-2Qnrl)2cos21Tr2 1 N2 = (- 2 Qn (A.l4) r 1)2 sin 21Tr 2 are independent random variables, normally distributed with zero mean and unit variance. 3.6.2 Model verification The final stage in the process of distribution selection and parameter estimation should be model verification. 31- 44. Structural Reliability Analysis and Prediction, Third Edition is a textbook which addresses the important issue of predicting the safety of structures at the design stage and also the safety of existing, perhaps deteriorating structures. A practical number is generally considerably less than the number of basic random variables. Reliability theory also has a role to play in the assessment of existing structures, particularly when structural damage has occurred as a result of accidental loading, or when a structure is being assessed for a radical change of use. SOLUTION STRATEGY Consideration will be restricted to response in a single mode. Experience shows that an expansion based on the mean point should not be used. E.: The Spectral Density for Ocean Wave Forces. [12.34] Rosenblatt, M.: Remarks on a Multi- Variate Transformation. [3.7] Ferry Borges, J. Use at order statistics: The graphical method discussed above is in fact a simpl.'! application of order statistics. J.: Variability in the Strength of Structural Steels - A Study in Structural Safety. Control of the second type is clearly more difficult to achieve since the sources of possible errors are almost unlimited. Example 3.5. Consider an axially-loaded reinforced concrete column, a cross-section of which is shown in figure 3.12. NewnesButterworth, London, 1979. 3.6 ESTIMATION OF DISTRIBUTION PARAMETERS 59 It cannot be emphasised too strongly that the blind application of statistical procedures can lead to very misleading results and that an initial inspection of the available data should always be undertaken before any formal calculations are made. It can now be seen that if the values x* were to be used as the design values xd in a deterministic level 1 design calculation, the resulting structure would have a reliability index {3 and a reliability fH = 1 - (P(~- (3). In general, the value of a i depends not only on the parameters of the random variable Xi' but on the values of the parameters of the other random variables, on the value of ~ and on the nature of the limit state function. ASCE, EM6, Dec. The problem now is to estimate suitable numerical values for the parameters of these distributions using available data. Report NBSIR 74-602, 1974. 1.25 J1xiJ10 +-__~________-+- L__________~__~_ 0.8 1.0 1.2 Figure 12.12h. These models are selected or devised in such a way as to embody those features of the physical quantity that are essential for the analysis of the practical problems being considered. Attention will be restricted here to the modelling of continuously distributed as opposed to discrete quantities. Topics such as the statistical theory of extremes, methods of parameter estimation and stochastic process theory are introduced in later chapters as and when they are required. 3) Determine a better estimate of z* from equation (12.78). Let the variable of interest X have a probability density function fx with unknown parameters o= (° ,° 1 2 , ..• , Ok) that are to be determined. The problem is of a different nature. It is reasonable in some situations to expect that a given random variable can be considered normally distributed as a good approximation, but often such an assumption is quite unreasonable. ASCE, J. It must therefore be modelled as a random variable. (Michael John), 1940. K.:Discussion on [12.25). is the square root of the relative velocity variance at the level of mass i. Ingenieur- und Architekten-Verein (SIA), ZUrich, 1981. 34/13/4047, LNEC, Lisbon, Feb. , xn) is the joint probability density function for the n variables Xi. Note that the integral is over the failure region, denoted w f (see chapter 5.2). 3. H. The joint density function fxx can be derived by the so-called convolution integral 164 10. Show that for this distribution, these estimators are the same as those obtained by the method of moments. D.: Methods for Statistical Analysis of Reliability and Life Data. Let the failure surface be linear and let the basic variables be normally distributed. 232 12. It should be noted, however, that unless the standard of inspection is high the probability that the dimension D will exceed the specified tolerance may not be negligible. 'X I =-00 fx X (xl,xl)fx X (~-xl,x--xl)dxldxldx 1 1 2 2 (10.10) (10.10) can be written in a more convenient form by the substitution + ~~ '·X=-oo \ 'w x = xl + x2 X2fx1x,,(x'Xl)fx2x2(~-x'X2)dwdx (10.11) where the domain w in the xl x 2 -plane is shown in figure 10.2. In conclusion the procedure for evaluating the distribution function F:E: for the maximum value of the stochastic process {X(t)} = {Xl (t) ----------~~----------~X1 Figure 10.2 + X 2 (t)} in the time interval [0; TJ is 165 10.2 THE LOAD COMBINATION PROBLEM x and fx 2 X2 (1) Calculate fx (2) Find !lxCO = E[N~ (3) Find an approximate expression for F::; from (10.7) 1'"1 ml for the two processes {Xl} and {X 2 } by evaluating the integrals in (10.11) Step (2) above can only be performed exactly for special density functions. = 25 N/mm2 Pi = 0.16. If, for example, the standards of quality control used in the manufacture of a structural material change, one would expect to see some change in the probability distribution function of that variable. John Wiley & Sons, 1974. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES Whether X may be regarded as a log-normal random variable in any practical situation in which X is the product of a number of random variables depends on the circumstances. Illustration of continuous time-varying load. A clear distinction must be made, however, between a histogram or a relative frequency diagram on the one hand and a probability density function on the other. Columbia University Press, 1958. Imperial College. It is not intended as a critical review of the reliability of jacket structures which is a subject beyond the scope of the present text. In practice, with detailed knowledge of the structure being designed or checked, it is often possible to reduce the number of safety checks significantly. ft 1T a·r,l. , Sm) equation (7.2) can be generalised and the probability of failure calculated if the corresponding joint probability density function is known. The contribution of the geometrical variables to the total uncertainty is negligible. vxCO'T (10.6) The left hand side of (10.6) is equal to 1 - F:=: (0, where F:=: is the distribution function of the maximum value of the stochastic process {X(t)} in the time interval [0; TJ. B.: Probabilistic Approaches to Design. The relatively simple structural examples given in this chapter and in chapter 6 are for the purpose of explaining the methods. [12.20) Kim, Y. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES In essence there are two types of estimates for distribution parameters that can be obtained - point estimates and interval estimates. , gdm)' etc. 3rd International Conference on Structural Safety and Reliability, Trondheim, 1981. Hasofer, A. [12.26] Marshall, P. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES with " ,.y = kr(1 +-) 13 (3.26) and Uy = k(r(1 + f) - r2 (1 + t))2 I (3.27) Comparisons of the type I maxima and type II maxima distributions with the normal and lognormal distributions are shown in figure 3.4. The random variables in each case have the same mean and standard deviation, namely 1.0 and 0.2. 3.4 MODELLING OF RESISTANCE VARIABLES - MODEL SELECTION 3.4.1 General remarks In this section some general guidelines are given for the selection of probability distributions to represent the physical uncertainty in variables which affect the strength of structural components and complete structures - for example, dimensions, geometrical imperfections and material properties. Let the failure function f :Rn"R be given by (6.21) and the corresponding safety margin The reliability index !3 is then simply equal to (6.22) i.e. the same value as determined on the basis of the reliability index !3 as defined by equation (5.9). 41 3.3 ASYMPTOTIC EXTREME-VALUE DISTRIBUTIONS +----~ 0, k > 0 (3.16) where the parameters u and k are related to the mean and standard deviation by Ily = ur(l-l/k) with k> 1 (3.17) with k> 2 (3.18) 1 Uy where r = u[r(l- 2/k) - r2 (1-1/k)]2 r is the gamma function defined by (k) = \ ~ e-uuk - 1 du (3.19) • 0 It should be noted that for k ..; 2, the standard deviation uy is not defined. RELIABILITY THEORY AND QUALITY ASSURANCE 246 increasing gross error magnitude }1R.opt(g) --i 130 110 I I I 100 I 90 I 120 I I I I I I 80 70 ------ l.0 0.5 0.1 Figure 13.2 increasing gross error magnitude 2000 ----- -1 Gross errors considered in choice _ _ _ of partial coefficients I I ____ Gross errors neglected in choice of partial coefficients ~-=--=--=--=--=-=-==-""'-=::::~ 1500 1000 ..... BIBLIOGRAPHY [9.1) Crandall, S. ROBERT E. Tang: Probability Concepts in Engineering Planning and Design. Most loads and other structural design parameters are rarely known with certainty and should be regarded as random variables or stochastic processes, even if in design calculations they are eventually treated as deterministic. The last three methods are suitable for computer application.)2 (3.39) 2 j=1 1 Assuming further that the various R j are also identically distributed normal variables, N(100, 20) with units of kN, and that re= 500 kN, then ilR = 500 + 12 X 100 = 1700 kN and uR = 69.28 kN Since R is also normally distributed in this case, the value of R which has a 99.99% chance of being exceeded is thus ilR + -1 (O.OOOl)UR = 1700 - 3.719 X 69.28 = 1442 kN This total load-carrying capacity corresponds to an average load-carrying capacity of (1442 - 500)/12 = 78.5 kN for the individual reinforcing bars, i.e. only 1.07 standard deviations below the mean. BECK, PhD, is an Associate Professor in the Department of Structural Engineering at the University of São Paulo, Brazil. + -1 (Pft))2 = I m j=1 wj(llj(r) -ll t)2 (11.41) 196 11. Let us restrict our attention to a single property, the dynamic yield stress, U yd ' determined at a controlled strain rate of 300 micro-strain per minute and defined as the average height of the stress-strain curve between strains of 0.003 and 0.005, i.e. u yd = 1 ~E=0.005 0.002 uy(E)dE (3.29) E=0.003 where Uy(E) is the dynamic yield stress at strain E. Substituting in equation (4.32) gives p-q, (f - Qn(:S R j~~S : ~) ') vi Qn«Vit + I)(V~ + 1)) (4.36) Convenient analytical expressions for Pf do exist for some other combinations of distributions of Rand S. Coastal Eng. This sample, although generated numerically, may be treated in the same way as any other statistical sample. This information can be gained by experience and by the occasional level 2 analysis. Let the set of correlated random variables X = (Xl' X 2 , X 3) have the means E[Xj = [2, 4, 6] and the covariance matrix Show that the random variables Y = (Y I' Y 2' Y 3)' where YI] = ~ [21 [Y 2 Y3 1 0] [Xl] X -y12 yI2 -y'3 yI2 y'3 2 X3 are uncorrelated with the covariance matrix 6.3 CORRELATED BASIC VARIABLES In this section it is shown how the Hasofer and Lind reliability index {3 can be calculated for structures with correlated basic variables. A level 2 method is therefore a method of design or analysis which in its simplest form comprises a check at only a single point on the failure surface, as opposed to level 3 methods where the probability content of the entire failure region is evaluated. 2nd Ed. 1971. For this reason the most uniform standards of reliability can be obtained over a range of different structures by using design values dd and id of the geometrical variables related to the specified values, as follows (11.10) (11.11) where 6 d and 6 i are additive safety elements. Examples of the latter are dimensional changes arising from temperature effects and differential settlement. Example A.!, Let a = 55 = (8 X 391 - 3) = 3125 and m = 2 26 = 67108864. Wiley, N.Y., Vol. As in the case of resistance variables, the process consists of three distinct steps • precise definition of the random variables used to represent the uncertainties in the loading • selection of a suitable type of probability distribution for each random variable, and • estimation of suitable distribution parameters from available data and any prior knowledge. [2.6) Feller, W.: An Introduction to Probability Theory and its Applications. If the applied load is P, failure occurs when C7T 3 ED4 P = Pc = 64U or re-arranging, when 1 D = (64L 2 p)4 C7T 3 E Taking logarithms to the base e gives £n(D) = '41 (£nP + £n64 + 2QnL - £nC - 3£n7T - £nE) Replacing D, the actual diameter, by 8, the variable diameter at which failure will just occur, and using the rules of expectation and variance for linear functions gives 1 E[£n8) = '4 (E[£nP) + E[£n64) + 2E[£nL)- E[£nC) - 3E[£n7T) - E[£nEI) and 1 Var[£n8) = 16 (Var[£nP) + 4 Var[£nL) + Var[£nC) + Var[£nEI) But from equation (4.33) Var[£nP) = £n[V~ + 1) = Qn[0.25 2 + 1) Similarly, Var [£nC) = 0.00995, Var[QnE) = 0.0606 = 0.00062 and Var[£nL) = 0.02225. Y., 1954. 1978. , n are line~ functions of Xl' ... In fact data banks for this type of information exist in many industries. GENERAL For simulation studies and Monte-Carlo analysis use has to be made of long sequences of random numbers (generally pseudo-random numbers). The development of the close bounds discussed in chapter 8 has brought the goal of complete system reliability analysis in sight, but it would be irresponsible to pretend that, with the present state of knowledge, the problems of undertaking a complete analysis of a complex structure involving many failure modes are not formidable. 1964. Includes lxi bliograIilies and index. It is therefore clear that any structural member whose strength is a linear function of a number of independent random variables may in general be considered to be governed by the normal law . Because there is an infinite number of sets of (n -1) values xd which will give the same design, the problem facing the code writer is to select the »best» set of values xd . These are typical of the questions that must be asked in any realistic load modelling problem. CIRIA Report 57, March 1976. It should be clear from the preceding arguments that a procedure of random sampling and testing of, say, reinforcing bars at a construction site and attempts to fit a standard probability distribution to the data will not lead to a sensible outcome. It will be re- called that the probability distribution function for the random variable Xi was derived in section 3.2. One estimate of the cumulative distribution function Fx (xi) (Le. the particular value of Fx for X = Xi) is thus i/n, but preferable estimates are i/ln + 1) or (i -1/2)/n, since for most distribu- tion types they can be shown to be less biased. RANDOM NUMBER GENERATORS 3. kilogrammes interpreted as Newtons - Error in detailing · .. is the sensitivity factor for the ith resistance variable, ,I IIS,i is the sensitivity factor for the ith loading variable, lIR and as are estimates of the sensitivity factor for the composite variables Rand S in the limit state function R - S = 0, a R ,I. 81 Chapter 5 LEVEL 2 METHODS 5.1 INTRODUCTION In chapters 1 and 4 an introduction to the fundamental concepts of structural reliability is given and the so-called level 2 methods are briefly mentioned. 4, 1979. Such a procedure is discussed in section 11.4.3. Before this, we shall consider an approximate direct method for the evaluation of partial coefficients. The corresponding safety margin is (5.17) and 5 f.iM = 20 -"2' 4 = 10 kNm a2 M = 4 + 25 . RELIABILITY OF STRUCTURAL SYSTEMS will result in different loads (stresses) in the single elements. The mean and standard deviation of Yare: J.l.y e 1 + (k-e)r(l +7f) (3.23) and Uy = (k -e)(r(l + F) - r2(1 + t))2 1 (3.24) The type III minima distribution (3.22) is often known as the 3-parameter Weibull distribution and has frequently been used for the treatment of fatigue and fracture problems. Note that this probability is not fT(t)dt. International Association for Bridge and Structural Engineering. Academic Press, N. The penalty to be paid for using the simplified design rules is some increase in materials usage. It can normally be assumed that these functions have been fully tested for random behaviour. Load and resistance parameters clearly require different treatment, since loads are generally time-varying. and SchQaf, S. It is clear from figure 10.1 that the maximum values of PI (t), P2(t) and PI (t) + P2(t) during the reference period need not appear at the same instant of time. The reliability of the structure may then be expressed as tR = 1- Pf = 1 ~ ~~ ... II, 1966. Methuen, 1964. This process is illustrated in the following simple example. Method of maximum likelihood: This method is generally more difficult to apply than the method of moments, often involving iterative calculations, but maximum likelihood estimators of distribution parameters can be shown to have a number of desirable properties [3.11). Figure 3.5 shows two sets of 100 observations of the thickness T of reinforced concrete slabs having a nominal thickness of 150 mm, which illustrates this point. This is illustrated in example 6.8. Example 6.8. Consider again the beam analysed in example 5.5. In example 5.5 the reliability index {3 was calculated solely on the basis of second order moments for the relevant basic variables, namely the load P, the modulus of elasticity E and the moment of inertia 1. E-Book Content Palle Thoft-Christensen Michael 1. As is well known in other branches of engineering, anyone of a number of different empirical models may often be equally satisfactory for some particular purpose, e.g. finite-element versus finite-difference approaches. 245 13.3 INTERACTION OF RELIABILITY THEORY AND QUALITY ASSURANCE Let the initial cost of the structure be given by (13.2) where a is a constant, and let the consequential cost of failure, should it occur, be (13.3) The probability of failure, given a gross error of magnitude g and a model uncertainty of magnitude k, may be expressed as Pflg, k = P(gR - kS';; 0) (13.4) and the expected conditional total cost as (13.5) Assuming that the model uncertainty K can take the following discrete values with probability mass p(ki) ki 0.4 0.5 0.6 p(ki) 0.2 0.6 0.2 Table 13.3 the expected total cost, given a gross error g, E[C T Ig], is E[CTlg] = I 3 (CI + cfPflg)p(k) (13.6) i=l The expected total cost given that there is a gross error g with a probability p(g), and no error (g = 1) with a probability 1 - p(g) is thus E[CT(g») = I 3 (CI i=l + cfPflg)p(k)p(g) + 3 .2.--' (C I + cfPf)P(k)(1 - p(g» (13.7) i=l Substituting for c I and c f from equations (13.2) and (13.3) gives E[CT(g)] = Variable 3 3 i=l i=l .1.: aJ.!R (1 + ~Pf Ig)p(k)p(g) + IaJ.!R (1 + ~Pf)p(k)(l-p(g» J.! a/J.! R - 0.07 S 100 0.10 a = 10 cost units ~ = 20 Table 13.4 (13.8) 13. Map this Gumbel distribution on a normal distribution as shown above and determine the expected value III so that (3 = 3 (solution III = 161'10-7 m 4). A.: Probabilistic Aspect of Ocean Waves. & Fiessler, B.: Two Applications of First Order Reliability Theory for Time-variant Loadings. Other sources of uncertainty may have to be assessed subjectively. [4.10] Freudenthal, A. If we now obtain a random sample of size n from a known type of distribution function Fx' but with unknown parameters e, the cumulative frequency distribution for the sample can be ex- pected to plot as a straight line if the appropriate plotting scale is used. [4.2] Baker, M. BIBLIOGRAPHY 65 [3.10] Lieblein, J.: Efficient Methods of Extreme-Value Methodology. If the variables ues of the variables at which failure is most likely to occur (if this event were to happen), i.e. i = 1,2, ... U.S. Department of Commerce, National Bureau of Standards. This results in the mapping of the limit state failure surface given by g(x 1 , x 2 ' ... (o~ + 0~)2 (4.26) where is the standard normal distribution function and J.lS' J.lR' Os and oR are respectively the means and standard deviations of random variables Sand R. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES Fx(x) 0.998 0.995 0.99 0.98 / 0.95 /' Mill »W» 0.9 50 mm plates 0.8 0.7 0.6 0.5 e/ ", 0.3 IJ" 0.2 / /' 0.1 /. Q, G, the yield stress of the steel E y ' and the model uncertainty Xm affecting the plastic moment of resistance of the section are assumed to be normally distributed random variables, with the parameters given in table 11.1. The yield stresses at the plastic hinge positions A, Band C are assumed to be the same and the geometrical variables are assumed to have no uncertainty. Some general guidelines are given below. For practical reasons it is generally necessary for the user of a level 1 code to work with specified valu~s of all the design variables rather than with actual characteristic values, some of which will not be known at the design stage. For a given structure and location, the building process can be divided into two distinct stages: 1) preparing a precise deterministic specification for the structure, and 2) building the structure and checking that the specification is met. The Integrity of Offshore Structures. , n are normally distributed. , n I (11.23) Xcorresponding to the design X are all normally distributed, then the set of values x* are the val- we obtain the set of values x* for the original basic variables point z* . '" aRa R ,1. In comparison with the idealised models used for calculation purposes, the actual behaviour of most structures is extremely complex and there is a tendency, as more research is undertaken and more becomes known, for the design procedures set out in structural codes to become increasingly lengthy and involved. Such findings are of considerable value in the planning of quality assurance schemes, but this will not be discussed here. For example, column 3 shows the effect of setting 'Ym l = 1.0, 'YfQ = 1.5 and 'YfG 1.13 (given here as a weighted average of 'YfGl and 'YfG2)' and thus effectively eliminating 'Y ml from the code. BIBLIOGRAPHY [A.l] Box, G. load fracture - f - - - - - - - - - - - - displacement Figure 7.1 load p - - - -.~--------!~ displacement Figure 7.2 115 7.3 FUNDAMENTAL SYSTEMS brittle element ductile elemen t Figure 7.3 7.3 FUNDAMENTAL SYSTEMS As mentioned in the introduction there are two fundamental types of systems, namely series systems and parallel systems. R.: Reliability Engineering Design. The basic methods are • the method of moments • the method of maximum likelihood • various graphical procedures • use of order statistics. For simple problems, there is clearly no difficulty in finding a suitable form for f, but care should be taken in situations when some of the loads act in a resisting capacity (e.g. with loads resisting, as opposed to causing, overturning). It arises when the parent distribution is of the form: 43 3.3 ASYMPTOTIC EXTREME-VALUE DISTRIBUTIONS with X? Nevertheless, a number of general rules apply. McGraw-Hill, N.Y., 1970. Royal Institute of Naval Architects, 1967. FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY 80 BIBLIOGRAPHY [4.1] Ang, A. Institution of Civil Engineers, Part 1, Vol. Consider a structure subjected to a random time-varying load Q having a specified nominal magnitude qsp. , x*) and where !lx' 1. [12.8) The British Ship Research Association: A Critical Evaluation of the Data on Wave Force Coefficients. and 1 ax. However, for the sake of completeness, the technique will be briefly discribed. 3.5.2 Choice of distributions for loads and other actions We now consider the process of defining appropriate random -.ariables and their associated probability distributions to model single loads and other actions. 2 £,.; I I I]] I (21T)2 ICI 2 i,j=l where i = (xl' x 2 ' _ ... and Walker, S.: Dynamic Analysis of Offshore Structures. 1979. 56 3. It will not have escaped the attention of the reader that the modelling of loads and actions re- quires a certain degree of subjective judgement. Level 2 methods provide a powerful set of tools for tackling a wide range of practical problems. Methods for doing this are briefly reviewed in section 3.6.2. 3.6.1 Techniques for parameter estimation This is a large subject in itself and only a brief description is possible here. McGraw-Hill, N.Y., 198!. Note that PXII x 2 is a mass function in (2.65) and fXII X2 a density function in (2.66). If you are author/publisher or own the copyright of this documents, please report to us by using this DMCA report form. This has enabled important conclusions to be drawn about the relative importance of geometrical imperfections in governing failure. Reliability analysis should not, however, be thought of as an isolated discipline as it is closely related to the theory of statistics and probability and to such fields as operations research, systems engineering, quality control engineering and statistical acceptance testing. H-S. In level 2 methods a number of idealizations compared with the »exact» level 3 methods are made. 1. Usually, there is hardly enough information to be confident about the marginal distributions and the covariances. Under §54 of the German Copyright Law where copies are made for other than private use, afee is payable to "Verwertungsgesellschaft Wort", Munich. 3.4 MODELLING OF RESISTANCE VARIABLES - MODEL SELECTION 49 From the preceding example it is clear that there are many sources of physical variability which contribute to the overall uncertainty in the yield stress of a grade of reinforcing steel. and Evandt, 0.: Experiences with Failures and Accidents of Offshore Structures. [4.6] Elderton, W. Only the most frequently used extreme-value distributions will be referred to here. This cannot be avoided. The moment estimators for IJ. In section 5.4 the reliability index {3 was defined in the following way. [3.15] Sentler, L.: A Stochastic Model for Live Loads on Floors in Buildings. R.: Dynamics of Marine Structures. The upper bound is obtained by changing the domain of integration in the first integral on the right hand side from w to WI and the domain of integration in the second integral on the right hand side from (.oJ to w 2 ' where WI and w 2 are shown in figure 10.3. Clearly, an upper bound of !lx(O is then obtained =r~ J_ao !Ix (x)fx 1 2 (~-x)dx+r~ J_ oo !Ix 2 (~-x)fx 1 (x)dx (10.12) where !Ix (x) and !Ix (~- x) are rates of upcrossings for the processes {Xl} and {X 2}. [13.11] Matousek, M. and Proschan, F.: Mathematical Theory of Reliability. , Yn are independent identically distributed random variables with finite mean Ily and finite variance u~, and if X = Y1 + Y2 + ... L.: Steel Reinforcement and Tendons for Structural Concrete. Control of the first type is aimed at reducing variability in the mechanical properties of structural materials and maintaining appropriate mean properties. Frequently, however, the data may be limited in number, in which case it is necessary to select the type of distribution a priori - from an understanding of the physical nature of the failure mechanism and/or from previous experience. Offshore Management: Permanently Located Offshore Structures - Jacket Sensitivity Study. It is convenient to describe the failure surface by an equation of the form (5.3) in such a way that positive values of f indicate safe sets of basic variables (the safe region) and non-positive values of f indicate unsafe sets of variables (the failure region), i.e. _ {> f(x) = 0 .;;0 when when x E Ws xE wf A 2-dimensional case is illustrated in figure 5.1. The function f: (5.4) wf'\ R is called the failure function. More complex and practical applications are discussed in chapters 11 and 12. = Cov[X.,1 X.]J i,j=1,2, ... The work is set out in the form of a textbook with a number of examples and simple exercises. For most structural reliability calculations, the analyst is concerned with obtaining a good fit in the lower tails of the strength distributions, but this may not always be important - for example, when the strength of a structural member is governed by the sum of the strengths of its components. Most of the examples in table 13.1 are of this type. 3.2 STATISTICAL THEORY OF EXTREMES In the modelling of loads and in the reliability analysis of structural systems it is necessary to deal with the theory of extreme values. [12.24] Malhotra, A. For problems with a small number of basic variables the same iterative method as used in the examples in chapter 5 can be used . , Ok' the right hand side of equation (3.46) is also a function of the same k parameters and t j may be expressed as (3.47) Using equation (3.46) to generate the first k moments tj we obtain k equations in the k unknown distribution parameters Or If we now consider a random sample of the variable X of size n with values (xl' x 2 ' •.• , xn) the equivalent sample moments are given by 1 n . There are many questions of this type and the nature of the system, the use to which it will be put and the consequences of failure dictate the type of analysis that should be undertaken. (10.13) ---------r~-------.-X1 / Figure 10.3 166 10. Journal of the Structural Division, ASCE, Vol. In general, loads and actions cannot be sensibly classified without a knowledge of the structure on which they are acting. Indeed, by their very nature, some potential gross errors or hazardous situations must exist which have not yet been recognised. Example 3.8. Consider a steel bridge loaded solely by a sequence of partially-laden vehicles. Therefore, in reliability analysis, single load variables imply no special difficulties. 0.05 0.02 0.01 0.005 ,- / 200 / / '" .-7 :Y / V ./ ~ _-y ~ r 0.4 ~ (/. Lind: An Exact and Invariant First Order Reliability Format. , Rn) and likewise the strength by a number of random variables S = (Sl' ... Equivalent scales in the original quantities Fx (x) and x can therefore be constructed. 48, 1977, pp. effect of ground-water pressure overlooked - Misinterpretation of geotechnical data - Computational error in analysis · .. From (5.21) approximate values for f.iM and OM determined by (5.22) (5.23) are 86 5. , Xn (6.3) The expected value E[Y] is then given by E[Y] = a o + I n j=l (6.4) ajE[X j] and the variance Var[Y] by Var[Y] = I n j=l a~ Var[Xj] + n I I j. If the load X(t) has a deterministic time-history given by x(t) = x.sin(wt) i.e. x(t) is a sinusoidally-varying force of known amplitude x., then it can be shown that xx which is a V-shaped distribution. The easiest starting point is to consider the probability density function fx of a random variable X as the limiting case of a histogram of sample observations as the number of sample elements is increased and the class interval reduced. The total design load effect is therefore given by (cf. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES 12 ilR = re and + E ilRi 12 U R = (3.38) j=1 1 (E u R. The probability Pe must be clearly distinguished from the probability p mentioned above. [13.16] SIA: Weisung fUr die Koordination des Normwerks des SIA im Hinblick auf Sicher- heit und Gebrauchsfiihigkeit von Tragwe.rken. All rights are reserved, whether the whole or part ofthe material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similiar meaos, and storage in data banks. Let us consider the practical problem of estimating the parameters of a single distribution function from a single sample of experimental data. 195 11.4. METHODS FOR THE EVALUATION OF PARTIAL COEFFICIENTS The target failure probabilities given in table 11.3 are for a reference period of 1 year, but should be treated as operational or notional probabilities and not as relative frequencies. Many structural en gineers are shielded from having to think about such problems, at least when designing simple structures, because of the prescriptive and essentially deterministic nature of most codes of practice. Values of the partial coefficients obtained by minimising the quantity S defined by equation (11.38), subject to the constraint given by equation (11.39), are listed in column 1 of table 11.4. The other columns in this table show the values of partial coefficients 'Ym 2 when other constraints are introduced. When fR,s is known, the probability of failure Pf can be calculated relatively easily from (7.2) by a suitable numerical technique or by simulation. The distribution function FR for the strength R of the system can then be derived by equation (7.3) if the strength of two elements can be assumed independent. His research interests include structural mechanics and structural safety. We shall first consider the problem of modelling physical variability and then turn to the question of incorporating statistical uncertainty. and Holand, I.: Risk Assessment of Fixed Offshore Structures - Experience and Principles. Part I: Steel for Reinforced Concrete. [2.5) Lin, Y. In the remainder of this section various formal procedures for the determination of partial coefficients are discussed. In general, the exact probability distribution for M will not be of any standard form, although it may be governed by the form of the probability distribution of the most dominant basic variable. Let X be a normally distributed random variable, having parameters IJ. This is in general difficult, but it has been done for some special stochastic processes. One important assumption that has been made is that the probabilistic models for loads and resistance variables are representative of events during a particular period of time - the life of the structure in the case of loads and other actions, and the period of construction in the case of 240 13. E.: Selection of Control Levels for Maximum Utility of Structures. 11.3 RECOMMENDED SAFETY FORMATS FOR LEVEL 1 CODES The safety format of a code is defined as the way in which the various clauses of the code regulate the degree of safety, or more generally the reliability, of structures designed to the code. , Xn (5.13) It is then easy to calculate the reliability index i3 as (5.14) and (5.15) where the last term accounts for correlation between any pair of basic variables. 48 3. For the present purposes it is sufficient to state that the maxima of time-varying loads can in most cases be represented by one of the asymptotic extreme-value distributions, with parameters estimated by one of the techniques given in section 3.6. 3.6 ESTIMATION OF DISTRIBUTION PARAMETERS It is assumed that the selection of the types of probability distribution for the various load and resistance variables has been made using the approaches and methods of reasoning discussed previously. 20 mm bars used instead of 40 mm - Misinterpretation of drawings · .. The distribution of the minimum Y of n independent and identically distributed variables Xi asymptotically approaches the form y-e F (y) = 1 - exp(- (--)~) Y k-e asn-> with y? 255 Appendix B SPECTRAL ANALYSIS OF WAVE FORCES 1. 1 when it is not. The probability that isp will be exceeded will generally be small and will depend on the standard of inspection. The increased use of such studies in recent years has meant that reliable library functions have been made available on most computer systems. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES Example 3.11. Models do not represent reality, they only approximate it. The selection of the probability p is to a large extent arbitrary but is influenced by the follow· ing considerations • characteristic values of loads and other actions are values which should rarely be exceeded, • characteristic values of material strength properties should normally be exceeded by actual properties, • the values of p should neither be so large nor so small that the values x k are not occasional· ly encountered, • it is often sensible to use previously adopted nominal values as specified characteristic values, xsp' The distinction between characteristic value and specified characteristic value (specified value) should be made clear. Failures of type A, and failures of type B resulting from an engineer's ignorance or negligence, are in theory preventable, but this requires an appropriate level of expenditure on education, training, design checking, quality control, inspection, maintenance, etc. Similarly, the user of a code should normally work with specified deterministic values of loads and other actions; it is the responsibility of the code writers to relate these values to the distributions of the actual loads and actions, and to recommend associated partial coefficients or other safety elements. [4.12] Kapur, K. Journal Eng. Royal INst. ...... and Johnson, N. When a reliability problem involves more than one time-varying load the problem is more complex (see chapter 10). 96 6. _J.L)2 p. INTRODUCTION The purpose of this appendix is to derive a relationship between the spectrum of water sur- face elevation S1)1) (w) and the spectrum of structural displacements Sss(w) for a typical multipile jacket structure in a given normal response mode. O. 2. It was shown that the relia· bility index {3 of Hasofer and Lind is failure function invariant in the sense that equivalent failure functions result in the same reliability index. The other part is to predict the strength or load-deflection characteristics of each structural component from the information available at the design stage. , n (11.24) where w f is the failure region. Both the form and the parameters of the probabilistic models for resistance variables discussed in chapter 3 are dependent on the standards of quality control and inspection that are in operation. GENERAL EQUATIONS OF MOTION Assuming that the structure can be idealised as a lumped mass system (see figure B.1), the equations of motion may be expressed in the well-known form m~s + cs ~s + ku s = pet) (B.1) L i + i -- 1 Figure B.1. 256 APPENDIX B. [2.3) Bolotin, V. The existence of this underlying population of nominally identical systems or components means that it is possible to interpret failure probabilities in terms of relative frequencies. [lOA] Madsen, H., Kilcup, R. Example 6.5. Let Xl and X 2 be random variables with means E[Xll covariance matrix ex = a 2 [; = E[X21 = J1 and the ~] exactly as in example 6.4. Further, let the safety margin Mx be defined by (6.23) The purpose of this example is to show that (6.20) and (6.22) yield the same value for the reliability index !3. An immediate field of application for the type of analysis described above is to assist with decision making in areas such as the selection of optimal fabrication tolerances and in the sched· uling of cleaning programmes for the removal of marine growth. However, distinction must be made between failures which are the result of ignorance within the structural engineering profession as a whole and those that occur because of ignorance or negligence by an individual or design team. 11.3.3 Treatment of geometrical variables Geometrical variables are of two main types - structural dimensions (e.g. the depth of a beam) and geometrical imperfections (e.g. the out-of-straightness of a column). In compiling the bibliography our approach has been to list only a selection of the more important works in each subject area, along with other works to which specific reference is made. Reports of the Working Commissions. It is, however, of the utmost importance to predict the existence of any adverse interactions between the components which may exist in the system but which have not affected the failure rates of the individual components when tested in isolation or under different conditions. The inverse equations are Xl = J1 + X 2 = J1 :f{- (a.JT+P Zl + a.,;r=p Z2) + .,;; (a.JT+P Zl - a.,;r=p Z2) Therefore, the safety margin M z can be written Mz 2 a~ (a l + a 2)Zl + ,j2 2 a.,;r=p(a l -a 2)Z2 -- a o + J1(a l + a 2) + ,j2 (6.24) 105 6.3 CORRELATED BASIC VARIABLES or (6.25) The reliability index of Hasofer and Lind defined by (6.20) is equal to the distance from the origin to the straight line (6.25), i.e. ~ = bo Ib 2 2 + b2 where b o = a o + p(a l + a 2) and b~ + b~ = a 2 (a~ + a~ + 2pa l a 2), so that y I a o + p(a l + a 2) ~=./2 2 aya l + a 2 2 + pal a 2 (6.26) (6.27) From (6.23) and Therefore, (6.22) gives the same result as (6.27). For example, in modelling the yield stress of steel, a simple log-normal distribution may often be used (see chapter 3), but on other occasions a mixed distribution model (see equation (3.34» would be more appropriate. [A.2] Hammersley, J. For such a random variable negative values cannot occur. Gross errors occur because of inadequacies in the standards of quality assurance - the process by which the various components of complete »building process» (mentioned above) are co-ordinated with the aim of achieving the design objective. Also note that maximum value of PI (t) + P2(t) is considerably smaller than the sum of the maximum values of PI (t) and P2(t). Methods of including statistical uncertainty were introduced in section 7 of ch~pter 3. Allowance should be made for the possibility of the occurrence of all recognised failure modes, e.g. shear, buckling, plastic collapse, etc., together with various modes of unserviceability. :..-+-: .. 127 (Part II). G.: Failure Modes of Offshore Platforms. It is reasonable to model this structure as a series system with two elements. However, the use of this equation implies a reliability analysis of the structure and if this is to be undertaken there is little point in following it with a level 1 safety check. Example 3.4. The yield stress of hot-rolled steel plates of a single nominal thickness and grade of steel, supplied by a single manufacturer, can be shown to be closely represented by a log-normal probability distribution (see equation (2.51», as illustrated by the cumulative frequency diagrams in figure 3.10. SIA 260, Schweiz. For new structures, the role played by reliability theory is in the preparation of the structural specification, either directly, by subjecting the proposed design to a reliability analysis or, indirectly, by using a code in which the partial coefficients have been assessed probabilistically. Turkstra's rule, p. Let the random variables YI and Y2 be linear functions of Xl' X 2 , ... and Bea, R. Secondly, even if the joint density function is known, or in the case of equation (4.42) the marginal densities, the multi-dimensional integration required may be extremely time-consuming. Therefore, Q B and MB are correlated although PI and P 2 are uncorrelated. The general approach in estimating the parameters of distributions of known type is to use sets of coefficients or weighting factors in conjunction with the order statistics to Jbtain estimates of the parameters. Applied Science Publishers, 1981. Let the random variable Y be defined by (2.85) where Xl' X 2 are random variables and aI' a 2 constants. Two types are given: those in which the structure fails in a predictable manner by one of a number of foreseen failure modes - here called type A; and those in which unforeseen failure modes occur - called type B. and Walker, A. Studies of the reliability of structural components designed to traditional codes typically show very wide ranges of reliability. These are not the same, since r * s at failure, see figure 4.2. It should be noted that Pf is not given by the area of overlap of the two density functions fR (r) and fs (s) in figure 4.1. This is a common misconception. RELIABILITY THEORY AND QUALITY ASSURANCE This information, which is passed from the designer to the contractor (or builder), is and needs to be of a deterministic nature. Each load or action can be described as • permanent or variable • fixed or free • static or dynamic These three independent attributes relate to the nature of the action with respect to • its variability in magnitude with time • its variability in position with time • the nature of the induced structural response Thus the load imposed by vehicles on a lightly-damped long-span bridge could be described as being variable, free and dynamic. [12.28] Morison, J. The constrained Fletcher- Powell technique can be used for this purpose. The fact that under very general conditions, the distribution function for the sum of a series of independent random variables tends to a normal distribution as the number of variables in the sum increases (refer to the central limit theorem in chapter 3) can be used to generate random numbers having a distribution which approximates very closely to normal. Bulletin d'Information Nos 127 and 128, 1980. and Castanheta, M.: Structural Safety. 3.4.2 Choice of distributions for resistance variables It has already been mentioned that unless experimental data are obtained from an effectively homogeneous source, formal attempts to fit standard forms of probability distribution to the data are hardly worthwhile. Alternatively, a formal goodness-at-tit test, such as the X2 test or the Kolmogorov-Smirnov test may be employed to ascertain the level of probability at which it is possible to reject the null hypothesis that »the random variable X has a particular distribution function with certain stated parameters». A more important result, however, is that even if the frequency of gross errors rises to say 5%, although, ilR,opt shows a marked increase for gross errors of moderate magnitude, the expected total cost given by 3 E[CT(g)lilR,opt] = I(cI + crPrlg, ilR,oPt)p(k)p(g) i=l 3 + I(cI + CrPrlilR,oPt)p(k)(l - p(g» (13.9) i=l is insensitive to the decision of whether or not to allow for the possibility of gross errors in calculating ilR,opt (see figure 13.3). We continue here with the problem of devising a suitable procedure for evaluating partial coefficients or other safety elements for a level 1 code. [12.5) Borgman, L. This assumes a measure of economy in the new code, but care has to be taken that these reliabilities are not too low. 10. This problem may be solved as follows. The shear force Q B and bending moment MB at the support Bare 1 Q B = 27(13 P l + 23 P2) 6 MB =27(4P I + 5P2) Q B and MB are random variables with the means and the variances aMB = 0.660 kNm The loads PI and P 2 are statistically independent. Holden-Day, 1969. FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY but owing to the nature of the end restraints, there is considerable uncertainty in its effective length L. Wind loading codes are perhaps the most advanced in this respect, e.g. [11.2). Danish Engi· neering Academy, Lyngby, pp. The simplest method is to check whether the sample data plot as a reasonable straight line on the appropriate probability paper. & Madsen, H. , n R (11.34) a S,l.=v'i-Yi-1 i = 1, 2, ... For these cases, 6 d and 6 i should be set to zero and the uncertainties in D and I should be allowed for by modifications to the partial coefficients on the other design variables. 13.4 QUALITY ASSURANCE The respective values of reliability analysis and quality assurance have been explored earlier in this chapter and have been shown to be entirely compatible. The theory is derived from the work of Borgman [B.1) and Malhotra and Penzien [B.3). The same is true of probabilistic models. The first step is to check the data for obvious inconsistencies and errors. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES uncertainty in the parameters within the distribution of X itself, in terms of what is known as the predictive distribution of X. [10.6] [10.7] Rackwitz, R. They are also questions that can only be sensibly answered when the precise purpose of the proposed reliability analysis is known. Depending on the distribution type, one, two, three or more parameters will be involved. These are ranked in order of decreasing importance in table 12.1. The fact that wind speed is dominant is not surprising because it is the main loading variable and because its extreme value is subject to considerable uncertainty. Example 3.13. The code replaces an earlier British Standard, BS 153 [11.1] and was developed mainly for the purposes of incorporating technical improvements in many of the design clauses; but at the same time the opportunity was taken to rationalise the safety provisions and to change from a permissible stress to a limit state approach. n 3. = y'l-v'J=! i = 1,2, ... In some examples, the parameters of the probability distributions used in the calculations have been chosen quite arbitrarily or in such a way as to demonstrate the calculation procedure with maximum effect. (~ is obtained by ignoring the damping term C~s). A uniform random number generator is one which generates successive independent realisations u j of a random variable U having a rectangular density function, usually in the interval [0, 11, i.e. (A.l) elsewhere giving (A.2) u>1 The mean and standard deviation of the random variable U can be shown to be Ilu = 0.5 au = 1/12 250 APPENDIX A. 4) Repeat 2) and 3) to achieve convergence. Longman, London, 1975. FUNDAMENTALS OF PROBABILITY THEORY It is important to note that independent random variables are uncorrelated, but uncorrelated variables are not in general independent. and Penzien, J.: Nondeterministic Analysis of Offshore Structures. First, there is almost: never sufficient data to define the joint probability density function for the n basic variables. In the remainder of this chapter we shall concentrate on the question of choosing suitable safety formats for structural codes and on the calculation of partial coefficients. If the value of Cov[X I , X 2] is numerically large but negative, then the values of Xl tend to be large when the the values of X 2 are small relative to their means, and vice versa. , k for the k unknown distribution parameters OJ may be obtained by equating the moments of X, t, and the sample moments m, i.e. (3.49) j = 1, ... A failure surface is a surface dividing the basic variable space into two regions namely a failure region w f and a safe region ws' The failure region contains all realizations of X that would result in failure, and the safe region contains all realizations of X that would not result in failure. me However, the required reliability is 0.9999. The problems that need to be considered here, however, are of a different nature. + Yn , then as n -> 00 P(a < X -nil Uy for all a, {3 (a Vny n (3.40) .;; (3) -> ({3) - (a) < (3), and where is the standard normal distribution function. , fn) of the random vector X = (Xl' X 2 , ... To take the simplest case, although a permanent fixed load is considered to be an action which does not vary with time or in position, it must generally be classed as an uncertain quantity for the purposes of reliability analysis, since in general its magnitude will not be known. In this chapter the treatment will be ex· tended so that correlated basic variables can be included. Example 3.10. Example 6.6. To illustrate the importance of taking the correlation into account consider example 6.5 with a o = 0 and a l = a 2 = 1, i.e. with the safety margin Then, from (6.27) it follows that 1 ~=(1+p)-2~* (6.28) where ~* corresponds to no correlation between Xl and X 2 (p = 0). but Q B and MB are correlated. error in position of reinforcement Inspection - Gross defect not detected · .. E.: A Note on the Generation of Random Normal Deviates. [6.4] Rackwitz, R. APPLICATIONS TO FIXED OFFSHORE STRUCTURES 5 wind speed 7 winu speed rie1d stress 5 4 nlarine growth 3 3 1 2 0.75 1.0 Figure 12.12a. Therefore, knowledge of the distribution of only the maximum values of the individual loading processes gives insufficient information to evaluate the combined effect exactly. Exercise 3.4. Given that the total load on a foundation is the sum of n independent but identically distributed permanent loads Pi' show that the coefficient of variation of the total load is only 1/..[n times that of the individual loads. and {Xl}' {X 2 }, etc. ASCE, ST10, Oct. Both loads and imposed deformations give rise to sets of action- effects (often loosely referred to as load-effects) within a structure, e.g. bending moments and shear forces. In such cases, a check should be made that (4.39) where Xi is any resisting variable known to be active in the limit state under consideration. • Xn (Xl' Xx' ... New examples and end of chapter problems are also now included. W. Note that but for example 32 2. [4.7] Ferry Borges, J. This does not mean that the practical aspects of structural reliability theory have been overlooked - indeed, the theory would be of little value if it could not be applied. , x~, where x~ is the smallest value and xi is the ith largest value called the ith order statistic. For a time-varying load Q, the characteristic value qk is normally defined as that value which has a prescribed probability p of not being exceeded within a given reference period. Research Report No. 22, Solid Mechanics Division, University of Waterloo, Canada, 1973. 10.2 THE LOAD COMBINATION PROBLEM One of the fundamental problems in dealing with time-varying loads modelled by stochastic processes is connected with estimation of the probability that the stochastic process defined as the sum of the individual processes crosses a given barrier (threshold) during the reference period T. This occurs when the number of partial coefficients is constrained to some small number. This is consistent with the results obtained from example 13.2 and indicates that relatively more resources should be deployed on control, inspection and checking - Le. quality assurance. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES Provided that the variations in yield stress along each 600 m length of continuously rolled bar can be assumed to be small in comparison with variations in Uyd, the average yield stress for the cast may be expressed as = (.) =1- ~ "(J I yd n ~ yd (3.32) (J i~l where cast. Dept. It should be noted, however, that some random deviations from a straight line are to be expected, particularly for points at each end of the line. In the next section the reliability index is redefined, so that this problem is solved in a simple way. 11.3. RECOMMENDED SAFETY FORMATS FOR LEVEL 1 CODES 185 Design values of dimensions and imperfections: Typically, the standard deviations of geometrical variables are independent of nominal dimensions (e.g. for given site conditions, the standard deviation in the thickness of a 100 mm slab is likely to be about the Eame as that of a 200 mm slab; giving a reduction in the coefficient of variation for increasing nominal thickness). Thoft-Christensen), Aalborg University Centre, Aalborg, Denmark, 1978, pp. However, for deterministic design purposes it can be represented by a single specified constant. , Xn n YI = IajXj j=l (6.6) n (6.7) Y2 = IbjXj j=l It can then be shown that Cov[YI , Y2] = I n j=l ajb j Var[X j] + n I I j• n (6.8) ajb j COV[Xi' Xj] j Example 6.2. Consider again the problem solved in example 6.1. The covariance between Q B and MB can now more easily be calculated by equation (6.8) 13 24 23 30 Cov[QB' MB] = 27 . Unlike resistance variables, most of which change very little during the life of a structure, loads and other actions are typically time-varying quantities. Ignorance of phenomena such as fatigue, brittle fracture and the deterioration of concrete made from high-alumina cement are typical examples from the past of errors in design concept. This leads to a damping matrix c 11.. In general, equation (4.17) or (4.18) needs to be evaluated by numerical methods. [5.7] Thoft-Christensen, P.: Some Experience from Application of Optimization Technique in Structural Reliability. , n (5.1) and the covariances c1J.. See also [4.2]. However, for highly damped systems, it is sufficiently accurate to use the diagonal terms of C and to ignore the off-diagonal terms. Example 3.9. Consider the modelling of the asphalt surfacing on a long-span steel bridge. and Rackwitz, R.: Non-Normal Vectors in Structural Reliability. [5.5] [5.6] Gravesen, S.: Level II Safety Methods. Further assume that p, Q and m F are realizations of uncorrelated random variables P, L, and MF with Mp = 4 kN a p = 1 kN ML =5 m aL MMF Figure 5.3 = 20 kNm ~O m aMF = 2 kNm 85 5.3 LINEAR FAILURE FUNCTIONS AND NORMAL BASIC VARIABLES The maximum moment is Iml = pQ/2 and therefore, the following failure function can be used (5.16) Note that Q in this case can be considered a deterministic parameter because Or., Therefore (5.16) can be rewritten ~ 0 m. 239 Chapter 13 RELIABILITY THEORY AND QUALITY ASSURANCE 13.1 INTRODUCTION The first 12 chapters of this book are devoted to various aspects of structural reliability theory and its application to design and safety checking. & Leopoldson, U.:Scatter in Strength of Data of Structural Steel. Let us assume that this property can be measured with negligible experimental error and that all the reinforcing bars from a single cast of steel are cut into test specimens 0.5 m long and then tested. The corresponding density function fT is also shown in figure 3.5. For comparison, figure 3.6 shows data obtained from a real construction site. 5) and (4 .16) Under these conditions the reliability lit is the probability that the structure will survive when the load is applied, and is given by (4.17) From considerations of symmetry , it can be seen that the reliability may also be expressed as (4.18) c.g. l:)(~ndin l! Inornent Failure density I : ~ Area=l +-___ ~~~ Fi~ure 4 .1 " ~~ ~ S,r 73 4.3 STRUCTURAL RELIABILITY ANALYSIS Figure 4.2. Since in general it is not meaningful to have negative strengths, the lower limits of integration in equations (4.17) and (4.18) may in practice be replaced by zero. Journal of the Structural Division, ASCE, July 1969, Vol. For failure modes in which part of the permenent load acts in a stabilising or resisting. Lack of authority F. ever, will be in the reliability assessment of complete structural systems. , n, and these basic variables are normally distributed N(/.li' 0i)' then the following relationship exists, 87 5.3 LINEAR FAILURE FUNCTIONS AND NORMAL BASIC VARIABLES Pr = (-~) f3 = - I~O • Q and when a1 = 0.1' 111' The failure function is defined by 111 so that 1 100 1 pQ3 Q -192 ~ =0 or 6ei -113 p =0 The basic variables P, E and I are normalized Zl = (P -I1p)/ap ' Z2 = (E -I1E)/aE and Z3 = (I - 111)/a1 • In the normalized coordinate system the failure function is given by The design point is now given by Zj =~CXj = 3cx j • The unknown 111' cx I ' cx 2 and cx 3 are therefore determined by the following system of equations: 113(4 + 3cx l) + 1.5 cx 2)(1 + 0.3 cx 3) 111 = 6.10 7 (2 1 cx I =1{ 113 cx 2 = -k"1 (5.40) 10 7 (3 + 0.9 CX 3)111 where k is determined by the condition cxi + cx~ + cx~ = 1. For the yield stress mentioned above this transformation should map R+ on R and be continuous. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES In an analogous way, if the strength of a structure depends on the strength of the weakest of a number of elements - for example, a statically determinate truss - one is concerned with the probability distribution of the minimum strength. , ' Fjgure 3.12. 257 APPENDIX B. 70 4. f. In fact, the models are conditional upon or pre-suppose certain standards of design checking, quality control, inspection and maintenance. , xn taken at random from the variable X. However, this interpretation may not in practice be too helpful. McGraw-Hill,1970. 198 11. APPLICATIONS TO STRUCTURAL CODES XR is a model uncertainty associated with the particular form of the resistance function, Xs is a model uncertainty associated with the particular form of the load effect or action effect function, and where DR and Ds are sets of different dimensions. [6.7] Thoft-Christensen, P.: Introduction to Reliability of Offshore Structures. [12.6) Borgman, L. Conf., ASCE,1965. The other quantities C, the model uncertainty, and E, Young's modulus, may also be assumed to be lognormally distributed with the following parameters /lC = 0.9 Vc = 0.1 Assuming that there is no uncertainty associated with the diameter D, find the required value of D such that the column has a reliability of 0.9999 for a 50 year reference period. In many practical cases e may be zero (i.e. representing a physical limitation on, say, strength). Then the covariance of Xl and X 2 is denoted Cov[X I , X 2] and is defined by (see page 33) The ratio Px! where x2 = ax! COV[X I ,X 2] and (6.2) ax!' aX 2 aX 2 are the standard deviations of Xl and X 2 is called the correlation coef- ficient. Journal Str. The mathematical form and parameters of the log-normal distribution were discussed in chapter 2 (equation (2.51». However, it illustrates the principle that the models which are used under normal conditions, without the presence of gross errors, are no longer applicable when a gross 241 13.2 GROSS ERRORS error occurs. domestic premises used for public library - Need for specialist maintenance overlooked · .. When Cov[X I , X 2] is close to zero there is no linear relationship between Xl and X 2 . It is important to note that the same failure surface can be described by a number of equivalent failure functions. crack in weld - Accidental loading Use - Change of use without structural assessment · .. The random variable M = f(X) is then called a safety margin. Usually numerical integration must be used. It follows from equation (2.76) that (2.81) Therefore, for uncorrelated random variables Xl and X 2 we have (2.82) 34 2. Unfortunately, exact expressions for Vx en are only known for some special kinds of processes. The purpose of this is to illustrate the use of the various models and calculation procedures that have been described in earlier sections. Choose the set of partial coefficients r, so as to minimise the quantity S given by S =I m j=1 w/~ (Pfi(r), Pft) (11.38) Subject to the constraint I m m j=1 Wj Pfi(r) = Pft IW with j (11.39) = 1.0 j=1 and where is an agreed function of the quantities Pfi(r) and Pft' is the failure probability of the ith structural component designed using the set of partial coefficients r, is the target failure probability, W= (WI , ... Eidgenossische Technische Hochschule (ETH), ZUrich, 1981. However, this simple representation of the problem is rarely of much practical use in structural reliability analysis, so we shall not consider them here. If the total load effect in a member were to be determined from (11.15) 187 11.3. RECOMMENDED SAFETY FORMATS FOR LEVEL 1 CODES where r fG 1 and gkl are the values

of r fG and qk for the first of m permanent loads, qkl is the characteristic value of the first of n time-varying loads Qj' rfQl is the partial coefficient associated with the load Q1 when this load is acting alone, and c is the load effect function, implying a linear or, where appropriate, a non-linear analysis of the structure under the action of the factored loads, the resulting load effect S would be extremely conservative. , X n , can be constructed so that the corresponding covariance matrix C1' is a diagonal matrix, that is Cy = (6.10) According to well-known theorems in linear algebra such a set of uncorrelated variables can be obtained by the transformation Y=A?:X (6.11) where A is an orthogonal matrix with column vectors equal to the orthonormal eigenvectors of Cx' By this transformation (6.12) (6.13) The diagonal elements of C1" i.e. Var[Yi], i = x. Finally, it should be emphasised that these conclusions are based on the assumption that the various R j are statistically independent. Other chapters cover the reliability of structural systems, load combinations, gross errors and some major areas of application. In section 5.4 it was shown that the probability of failure Pf can be related to the reliability index {3, when the following two conditions are fulfilled (a) the failure surface is linear (b) the basic variables Xi' i = 1, 2, ... If the steel is to be supplied by a single manufacturer and very close control is exercised over the chemical composition of each cast, variations in aYd will be very small; but if the chemistry is not well controlled significant differences between casts can occur. Referring to chapter 2, this is given by (4.5) since, integrating by parts, ~: !itT (t)dt = [t!ltT(t)]~ + ~: tfT(t)dt = E[T] (4.6) provided lim [t!ltT(t)] = o. It also shows how a single time-varying load may be treated. Typically, q is taken to be between 0.95 and 0.99, corresponding to the 5% and 1 % fractiles of the variable E. Davenport, A. and a 2 are now determined as follows. We now consider the reasons for using partial coefficients as opposed to single safety factors or load factors. 2nd International Conference on the Behaviour of Offshore Structures, London, Aug. Depending on the form of the failure function g, there mayor may not be local minima (and local maxima) present. ~ fXl ,X 2 • • • • • Xn (Xl' X2 ' ... Sketch the density function for the peak magnitude 159 BIBLIOGRAPHY and calculate the probability of obtaining peak values greater than 5. Attention is focused on the development and definition of limit states such as serviceability and ultimate strength, the definition of failure and the various models which might be used to describe strength and loading. 2 2 By inserting (10.9) in Rice's formula (10.8) one gets vxm=\'~ x\'~ -' • • x=O '-Xl (=-00 -,. Kitaigordskii. Nevertheless, the total amount of computational effect is considerable because all the probabilities Pfi need to be re-evaluated for each adjustment to the partial coefficients r. On re-arrangement, equation (B.4) then becomes (B.8) The term (m + cb - p A) is the effective m~s matrix, Cis a damping matrix including com- ponents of structural and fluid damping, cfu is the contribution to the loading due to drag, - ;; p and cb up is the contribution to the loading due to inertia effects. Both are essential and it is helpful to distinguish their individual roles. [12.16) Hallam, M. VDOC.PUB Download Embed This document was uploaded by our user. fXn (xn)dx l dX 2 .•• dXn (4.42) f(X) .. Furthermore, this approach leads to a partial coefficient on every basic variable, which is too many for practical use in design. Suitable functions are: (11.40) and S2 = I m Wj(--I (Pfj(r)) j=1 where Il is the reliability index. In this section only point estimates will be discussed. The evaluation of 8" thus requires the solution of the set of k equations j = 1, 2, ... Fx may take on a wide range of form and depends on the nature of X(t) - i.e. whether X(t) is a deterministic or stochastic function of time, whether the load can assume both negative and positive values, etc. C. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES 40 --+---+-'---+---+=--c'-----t=----+---_ x o 2 3 4 Figure 3.2. The probability density function fxn = dd (Fxn) is shown in figure 3.2 for various values of n and with X distributed N(O, 1). To reo tain this relation for non-normally distributed basic variables also, it is natural to approximate a non-normally distributed variable with a normally distributed variable. which is flatter and 1 has less pronounced tails (platykurtic) than any of the component distributions fx /Ai n Bi· Furthermore, it is generally found that the density function fx IBi representing bars of a particular size considered over all manufacturers is highly positively skew. 38 3. In the situations described above only one single structural member with a single failure mode is treated. Struct., Washington, D.C., 9 - 11 April, 1969. , n. They are generally easy to identify. His main areas of research expertise are in structural engineering risk and reliability analyses, probabilistic modelling of engineering systems, corrosion and deterioration modeling, and investigation of structural failures. The word estimate is used in this context advisedly. J. Whilst basic data are available for the time to failure of electronic and mechanical components, no such information is available for structural components, because in general they do not fail in service. APPLICATIONS TO STRUCTURAL CODES Because of the time-varying nature of most loads, the problem of assessing the combined effect of a number of different loads acting on a structure has been seen so arise. G., Heaf, N. 5.2 BASIC VARIABLES AND FAILURE SURF ACES One of the first problems one has to solve before the reliability of a given structure can be evaluated is to decide upon which variables (quantities or parameters) are of relevance. A structural element is called perfectly brittle, if it becomes ineffective after failure, i.e. if it loses its load-bearing capacity completely by failure. 0.: Some Experience with the Rackwitz-Fiessler Algorithm for the Calculation of Structural Reliability under Combined Loading. For many structures, however, the probability of failure is insensitive to small variations in structural dimensions. To handle problems of this kind it is sometimes useful to consider from a systems point of view. Since M"'; 0 corresponds to failure, Pf may be expressed as Pf = P(M ,..; 0) = lim kIn (4.44) n-+~ where n is the total number of trials, and k is the number of trials in which f(xl ' x 2 ' .•• , xn) ,..; O. APPLICATIONS TO FIXED OFFSHORE STRUCTURES 12.5 SOME RESULTS FROM THE STUDY OF A JACKET STRUCTURE In the last part of this chapter which is based on [12.2], some results are given from the analysis of a typical deep-water jacket structure. Then the following relation between the probability of failure Pf and the reliability index {3 exists (see (5.33)) (6.34) where 'I> is the standardized normal distribution function. The Gumbel distribution is now transformed into a normal distribution with the mean J.1~ and the standard deviation up given by the equations (6.37) and (6.38). can be used. 1·"]. 3.5 MODELLING OF LOAD VARIABLES - MODEL SELECTION 3.5.1 General remarks The term load is generally understood to mean those forces acting on a structure which arise from external influences - principally the effects of gravity, and aerodynamic and hydrodynamic effects, e.g. structural self-weight, superimposed loads, snow, wind and wave loads. However, it is first necessary to introduce the important subject of the statistical theory of extremes which is of relevance to both load and strength variables. 4.3.3 Problems Reducing to the Fundamental Case In some simple situations, although Rand S may each be functions of a number of other random variables, it may be possible by means of appropriate transformations to reduce the problem to the simple form. No discussion of structural reliability theory is therefore complete without some consideration being given to these additional causes of failure and their possible treatment. It arises whenever the random variable of interest X is the sum of n identically distributed independent random variables, Yj , irrespective of the probability distribution of Yi' provided the mean and variance of Yj are finite. 13.2.2 Classification of gross errors Table 13.1 gives a general classification of the nature and sources of gross errors, along with some examples. An obvious way of calculating Vx vxCO = m is to use Rice's formula (9.34) E[N~ COJ = ~: xfxx(L x)dx (10.S) where fxx is the joint density function for the process {X(t)} and its derivative process {"X(t)}. Study No.3: Structural Re- liability and Codified Design. SPECTRAL ANALYSIS OF WAVE FORCES where m is a diagonal mass matrix Cs is a structural damping matrix k is a structural stiffness matrix Us is a vector of displacements of the various lumped masses - dus dt and where p(t) is a vector of wave loads Pi(t) acting on the lumped masses ,(Zi+ Zi+l)/2 Pi(t) = \ (B.2) dP(t) • (zi + zi-l)/2 Substituting for dP(t) from equation (12.37), equation (B.1) becomes m~ s +c~ +kus=c(~ -~)I~ -~ s I+cb(~ p -~)+pA~ ss ap s p s s (B.3) where - up is a vector of water particle velocities ca is a matrix involving parameters such as Cd' D, cb is a matrix involving parameters such as C m , p, A (see chapter 12) A is a diagonal matrix of cross-sectional areas. If, for the sake of simplicity, the load-carrying capacity of the column is assumed to be given exactly by: 12 R = rc + ~ '~R.I (3.35) j;l where rc is the load-carrying capacity of the concrete (assumed known) and R j is the random load-carrying capacity of the ith reinforcing bar at yield. Equations (11.27) and (11.28) and similar relationships for other types of probability distribution are only of direct use, when the values Ii are known. ,n (12.80) Finally, a caution about local minima. The vast majority of structural failures occur because of gross errors. The design clauses given in level 1 codes should be interpr8ted as a set of decision rules, the outcome of which can be modified by changes to a set of control parameters - the partial coefficients. The fluctuations in yield stress within each 600 m length are typically very small, i.e. in the order of 1 - 2 N/mm 2 • For each 600 m length Q, the spatial average yield stress yd is defined as uyd = t ~: u yd dQ u (3.30) The variations in Uyd from one rolled length to another are typically larger than the withinlength variations and are caused mainly by differences in the temperature of the ingot at the start of rolling and by a number of other factors. It can be shown that (6.3) The random variables are said to be uncorrelated if Cov[X I , X 2] = o. [4.9] Freudenthal, A. C.: Monte Carlo Methods. These are most conveniently generated using a digital computer. 0 (4.4) Expected life: A further property of a system or component which is of interest is the expected life or the expected time for which the system or component can be expected to operate satisfactorily. For example, the actual load-carrying capacity of a structural member, as opposed to its nominal capacity, may decrease or may not significantly increase if, for example, any change in r Q or r m results in the designer using larger diameter reinforcing bars which, in spite of having the same specified yield stress as the bars they are replacing, may have a lower mean 11.2. STRUCTURAL SAFETY AND LEVEL 1 CODES 179 yield stress (see figure 3.9). The probability of failure within any given time interval [t, t + Ii t] is the probability that the actual life T lies in the range t to t + Ii t and is given by (4.7) 69 4.2 ELEMENTS OF CLASSICAL RELIABILITY THEORY The average rate at which failure occurs in any time interval [t, t + Ii tj is defined as the failure rate and is the probability per unit time that failure occurs within the interval, given that it has not already occurred prior to time t, namely iHT(t) - iHT(t + Ii t) Ii t iHT(t) (4.8) The hazard function is defined as the instantaneous failure rate as the interval Ii t approaches zero. Figure 13.1, taken from [13.8], illustrates this point. Incompetence 20 G. The rate at which the sum tends to normality depends in practice on the presence of any dominant non-normal components. 6. The Monte-Carlo approach is to use appropriate random number generators (see appendix A) to generate independent sample values Xi for each of the basic variables and to determine the corresponding value of the safety margin M from (4.43) By repeating this process many times it is possible to simulate the probability distribution for M by progressively building up a larger sample. A further difference between electronic/mechanical systems and structural systems is that whereas most structural systems are))one-off)), all but the most complex electronic and mechanical systems are produced in considerable numbers and can be assumed to be nominally identical. Most electrical, electronic and mechanical components and systems deteriorate during use as a result of elevated operating temperatures, chemical changes, mechanical wear, fatigue, overloading, and for a number of other reasons. The reliability of a real structure is usually much more difficult to evaluate since more than one element (member) can fail and because there is possibility of more than one failure mode for the system. [13.12] Melchers, R. Example 4.4. A slender cylindrical column of diameter D is to be designed to carry a timevarying axial load P, the maximum value of whi"ch in any 50 year period may be assumed to be lognormally distributed with a mean of 250 kN and a coefficient of variation Vp = 0.25. GENERATION OF RANDOM DEVIATES The length of this sequence before repetition is approximately 2 26 - 2) = 224 = 16777216. 242 13. Netherlands Industrial Council for Oceanology. It is clear that the mathematical form of fx will depend on the particular subset of X, e.g.: Let Ai be the event [bars are supplied by manufacturer iJ B. Single time-varying loads: If a structure or structural component is subjected to only permanent loads G and one time-varying load Q, the load-combination problem does not arise. =j +r m 1m 11 (A.4) i.e. r 1 is the fractional part of the quotient (aio)/m and j1 is the integer part. , Xn) with expected v!lues E[X i], i = 1, 2, ... 58 3. Usually, an extemalload on a series system Figure 7.4 2 Figure 7.5 n 116 7. [12.14) Draper, L. When the loading of a single structural element is determined by a number of random variables R = (R 1 , ... It should be noted, however, that not all gross errors make a structure weaker - they can also make it unnecessary strong. This is best explored by means of an example. Div., 1974, pp. Lack of education C. A commonly used random number generator on, for example the CDC 6000 Series computers is rn + 1 = 186277 rn (modulo 2 48) This has approximately 2 46 "" (7.04 X 10 13) random numbers before repetition, a sufficiently large number for most purposes. It is widely recognised, however, that most structural failures occur for unexpected reasons and in ways that have not previously been encountered [13.2], [13.7], [13.11], [13.14], [13.17]. When applying this procedure over a number of codes for different construction materials, e.g. steel and concrete, a further constraint that should be considered is to make the partial coefficients on loads and other actions the same in each code, irrespective of construction material. +QnZn tends to normality as n -> 00, = IQnZj j=1 (3.42) following the central limit theorem, regardless of the probability distribution of QnZ j . The log-normal distribution is, however, used very widely in reliability studies. Uncorrelated random variables Y1 and Y2 are given by Y1 =..f!- (L + C) (6.29) with E[Y l] = 3y'2 m, E[Y 2] C = y = 16 1 [1.5 o 0 0.5 = v'2 m and the covariance matrix J (6.30) From (6.29) the random variables Land C can be expressed by Y1 and Y2 -Y1 L - 2 (Y1 + Y2) C Let k = =Y1 2 (Y1 -Y 2) (6.31) 6.415-10-4 and let the failure criterion be or 1 - 6.415'10-2 c2 .;;;; 0 (6.32) From (6.31) it follows that LC = 1. The use of the hazard function is in indicating whether a system or component becomes progressively more or less likely to fail per unit time as time progresses. I.: Analysis of Structural Failures. Setting (4.19) and (4.20) we obtain, respectively, the probability density function of the resistance R' of structures in which failure occurs and the probability density function of the load effects S' which have caused failure. 25, No. 11, 1954. , zz). In particular, such a distribution will behave poorly as a predictor of the occurrence of values of X outside the range of the sample obtained. For example, the variability in the yield stress of steel can be reduced by improved control on chemical composition and rolling conditions. Inconsistencies are often found to arise when the set of data has been obtained from experimental test programmes in more than one laboratory. In previous chapters, various aspects of structural reliability theory have been discussed, together with the problems of modelling load and resistance variables. Equation (B.3) may now be written in terms of i: and r as (B.4) with e= (cs - c) i: + ca i: I i: I (B.5) The right hand side of equation (B.4) is completely defined for a given wave frequency, containing terms involving only water particle displacement, velocity and acceleration. [13.7] Feld, J.: Lessons from Failures of Concrete Structures. T. If, and only if, the basic variables placed by tR X are statistically independent, equation (4.41) may be re- = 1 - Pf = 1 ~ ~ ~ .. , xn) and y = (YI' ... S.: An Examination of the Within-Cast Variability of the Yield Strength of Hot-Rolled High-Yield Reinforcing Bars. D.: The Art of Simulation. 1, Wiley, N. [12.30] Newland, D. To some degree nearly all loads could be considered to be variable, free and dynamic, but whether each is classified as such depends on the response of the structure to the loading. Den Private Ingeni0rfond ved Danmarks tekniske H0jskole, K0benhavn, 1979. If the values of x max are represented by the random variable Y, then Fy is the distribution function of the largest extreme load. equation (11.6)) are obtained from (11.13) (11.14) where 'Y fa and 'Y fQ are partial coefficients and gk and qk are characteristic values of the random variables G and Q, respectively. Experience has shown that the values of the partial coefficients are generally very insensitive to the form of the objective function used (equation (11.38)). For a = 0, one gets from (9.52) P(~ > 9) = 1 - 9) = e _.1 (9 -- 5)' 8 ax = 0.135 The sta~dard deviation for the derivative process can be calculated by setting E[N~ (9)] 10-3 • One gets = 0.046. 13.3.2 The effect of gross errors on the choice of partial coefficients The problem that must now be considered is whether knowledge that gross errors can occur during the processes of design and construction should affect the rational choice of partial coefficients for use in level 1 codes. and &2 for the parameters J.L and a 2 may be obtained from p. Transactions, ASCE, 1962, Vol. Provided a further set of conditions hold, the central limit theorem also applies to the sum of independent variables which are not identically distributed. {fXl (Xl)fx 2 (X 2) •.. Structural stability. ,X n) = (f.i l , ... The particular problems associated with the analysis of combined loading are discussed in chapter 10. No such list can, of course, be comprehensive. The code writer is free to choose as many partial coefficients or additive safety elements as is considered appropriate for a given code. 7.2 PERFECTLY BRITTLE AND PERFECTLY DUCTILE ELEMENTS It is of great importance for a structural system whether its elements can be considered per- fectly brittle or perfectly ductile. Some suitable distributions are given by Elderton and Johnson [4.6]. The principles of reliability analysis have been applied to a very large class of problems, ranging from the design of control systems for complex nuclear and chemical plant to the design of specific mechanical and structural components, as well as more generally in the field of electronics and aero-space. and Struct. which can be expressed as (3.64) where L (0 Iz) is the likelihood of 0 given the observation Z, and f~ (0) is the prior density of 0, before obtaining the data, and N is a normalising constant. 1.25 l.50 €. 248 [13.3] 13. et al.) Volume I, Tapir, Trondheim, 1977. APPLICATIONS TO STRUCTURAL CODES • specify appropriate quality control measures and acceptance criteria for the manufacture and fabrication of basic materials and components, • determine the parameters of the relevant models from loading data and from materials data obtained under the specified standards of quality control and inspection, • select a suitable safety format - the number of partial coefficients and their position in the design equations (i.e. the variables associated with partial coefficients), etc., • select appropriate representative values of all basic random variables (e.g. nominal, characteristic or mean values) to be used as fixed deterministic quantities in the code, • determine the magnitude of the partial coefficients to be used in conjunction with the above representative values to achieve the required standards of reliability. LOAD COMBINATIONS (10.9) where fx y. See also [3.11]. Lectures on Offshore Engineering (eds. Graff and P. The second step of selecting a suitable probability distribution for each random variable can rarely be made on the basis of sample data alone and as in the case of resistance variables physical reasoning must be used to assist in this process. For one and two-parameter probability distributions, estimates of the distribution parameters can then be obtained by drawing the »best» straight line through the plotted points either by 3.7 INCLUSION OF STATISTICAL UNCERTAINTY 63 eye or using a formal least-squares method. A better approach is to include the statistical 64 3. Journal of Industrial Aerodynamics, Vol. FUNDAMENTALS OF PROBABILITY THEORY Therefore, Xl and X 2 are not independent. It is also of interest to note that although equations (4.17) and (4.18) give identical numerical values for Pf they are in fact fundamentally different. In undertaking a reliability analysis, the engi· neer should take account of all known sources of uncertainty and should use this information to control the probabilities of structural failure and unserviceability within acceptable ranges. , Xn) are correlated the first step is to obtain a set of uncorrelated variables Y = (Yl, ... The effect of this will be to make the overall construction slightly less economic and the reliability of those structures designed to the code marginally more variable, for any specified standard of reliability. 0.: Load Combinations in Codified Structural Design. Provided equation (11.39) is satisfied, the effect of these additional constraints is to increase the deviations from the target failure probability Pft and to increase the average amount of material used when designing to the code. Div. This approach may not be too unconservative since any non-homogeneity in the data will tend to artificially enhance the variance. Conf. [l3.15] Schneider, J.: Organisation and Management of Structural Safety during Design, Construction and Operation of Structures. Equation (4 .14) gives the total probability of failure Pf as the product of the probabilities of two independent events, summed over all possible occurrences; namely the probability PI that S lies in the range x,x + dx and the probability P2 that R is less than or equal to x. Royal Society, A 237, 1956. The required diameter for a reliability of 0.9999 in a 50 year period is thus 117.3 mm. Ingb., MUnchen, 1977. This approach is followed below. P., Johnson, J. In general the engineer is likely to have only a relatively small sample of actual observations of X, along with some prior information obtained from a different source. Department of Civil Engineering Report SRRG/3/80, September 1980. StuPOC-V-5-6. FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY Reliability function: Typically, however, the probability of failure of a system or component is a function of operating or exposure time; so that the reliability may be expressed in terms of the distribution function FT of the variable T, the random time to failure. It is therefore more realistic to assume log-normal distribution in this case. In this case it is necessary for the quality control procedures specified by the code writers to be such that the actual characteristic strength of the material exceeds the specified strength by an appropriate margin or with a stated probability. Then, if the various R j are statistically independent, 12 E[R] = E[rc + 2~ R j] = rc + j;l 12 I j;l E[R j] (3.36) Var[R j] (3.37) and Var[R] = Var[rc + 12 I j;l 12 Rj] = I j;l i.e . This is the subject of chapter 13. [4.11] Haugen, E. APPLICATIONS TO STRUCTURAL CODES d Figure 11.1 (b) then the actual characteristic value d k and the specified nominal value d sp will generally be very close - see figure 11.1 (a). In Chapter 8 some important reliability bounds for structural systems will be shown, and for systems with unequally correlated elements some approximate methods of estimating the failure probability will be presented. Source Nature Example Design - Possible failure mode unrecognised · .. John Wiley and Sons, 1968. In general, the function f can take any form, provided that M ..; 0 corresponds to a failure state and M> 0 to a safe state. One obtains where for r 0) is then given by ~' F:=;CO= 1-e -2a~ o..;;~ 9) can be calculated by considering the cases a = 0 and a = 1. J.: Failure and Survival in Fatigue. It should be noted that the scales chosen in figures 3.10 and 3.11 are such that a logarithmic normal distribution plots as a straight line. , Xn) is chosen in such a way that a failure surface (or limit state surface) can be defined in the n -dimensional basic variable space w. Let x(t') be the magnitude of a single time-varying load X(t) at time t'. FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY o m ~M Figure 4.3. Illustration of the reliability index~. University of California, Berkeley, 1965. soft stratum not detected · .. If it becomes progressively more likely to fail then clearly action should be taken to replace the component or system at some stage or to minimise the consequences of failure. Start {3 p (Fp(x;))u (Fp(X;)))/fp(x;) _-1 up = 'P(3 Iteration No. 1 2 3 4 5 6 3.53 3.50 3.40 3.34 3.33 3.32 a1 -0.58 -0.52 -0.35 -0.24 -0.20 -0.18 -0.18 O!z -0.58 -0.78 -0.88 -0.94 -0.95 -0.96 -0.97 0!3 0.58 0.35 0.31 0.26 0.22 0.20 0.20 u~ 1.708 1.425 1.337 1.237 1.176 1.150 1.140 J.1~ 3.054 3.479 3.580 3.678 3.728 3.746 3.752 Table 6.1 110 6. If the safety margin M is non-linear in X f.iM and M aM = = (Xl' ... G.: The Dependence of Wind Loads on Meteorological Parameters. joint density function fx X X (xl' X2 ' ... The second part of the building process involves the transformation of the specification into physical reality - the structure - and checking that it is satisfactory. = y- 1), let (B.14) E-Book Information Year: 1,982 Edition: 1 Pages: 268 Pages In File: 266 Language: English Identifier: 978-3-642-68699-3,978-3-642-68697-9 Doi: 10.1007/978-3-642-68697-9 Cleaned: 1 Orientation: 1 Paginated: 1 Org File Size: 5,589,484 Extension: pdf Tags: Civil Engineering Toc: Front Matter....Pages I-XIIIThe Treatment of Uncertainties in Structural Engineering....Pages 1-11Fundamentals of Probability Theory....Pages 13-35Probabilistic Models for Loads and Resistance Variables....Pages 37-65Fundamentals of Structural Reliability Theory....Pages 67-80Level 2 Methods....Pages 81-93Extended Level 2 Methods....Pages 95-111Reliability of Structural Systems....Pages 113-127Reliability Bounds for Structural Systems....Pages 129-143Introduction to Stochastic Process Theory and its Uses....Pages 145-159Load Combinations....Pages 161-175Applications to Structural Codes....Pages 177-202Applications to Fixed Offshore Structures....Pages 203-237Reliability Theory and Quality Assurance....Pages 239-248Back Matter....Pages 249-267 These should be eliminated if possible. EXTENDED LEVEL 2 METHODS 6.2 CONCEPT OF CORRELATION The concept of correlation has already been introduced in section 2.9. Let Xl and X 2 be two random variables with the expected values E[X I] = J1x! and E[X 2] = J1X 2 . Courses on structural safety have been given at some universities for a number of years. In many structural reliability problems, however, the basic variables are best described by mixed distribution models for which the tests described above are not applicable. [13.17J Sibly, P. & B. Cambridge University Press, 1975. The last step is to check that the sample data are well modelled by the chosen distribution and parameters. .. The corresponding orthonormal eigenvectors vIand v 2 are determined by the equations [: :]',0[: and [-: _:] '2 0[:] resulting in v =f l (1 1 2 ' -1) and The transformation matrix -v 2 -fl 2 (1,1) A is therefore The »new» uncorrelated random variables given by Y= (Y l' Y 2) are therefore or The expected values are E[Y1] =vI2 2 (2 - V; E[Y2 1 = 3) (2 + 3) 1 =-2"v12 =tv12 and the covariance matrix OJ = [2 0] A2 0 4 Note that Cy can be calculated in the following way (6.14) 101 6.2 CONCEPT OF CORRELATION but Cy is of course easier determined by (6.14). and Gumbel, E. exp('1~Vi) (11.28) 1 where !lX. Some of these steps have already been considered in some detail, e.g. the modelling of load and resistance variables, and others, e.g. quality control procedures, are beyond the scope of this book. A preferable approach is to make use of physical "reasoning about the nature of each particular random variable to guide the choice of distribution. [9.4) Nielsen, S. [12.17) Hasselmann, K. MELCHERS, PhD, is a Professor in the Department of Civil Engineering at The University of Newcastle, Australia. But the characterization »perfectly brittle» can also be used in situations, where no real fracture occurs, e.g. when the element is deflected so much that it is ineffective in relation to a given loading. Equation (4.14) is best understood by plotting the density functions of Rand S, as shown in figure 4.1. It should be noted that Rand S must necessarily have the same dimensions (e.g. loads and load-carrying capacities, or bending moments and flexural strength). Thus (4.32) The properties of the lognormal distribution are such that if Y is lognormally distributed and X = QnY, then a~ = Qn[Vi + 1] (4.33) ~ 1 my = Ily exp(-2" a~) (4.34) and (4.35) where my is the median of Y and Vy is the coefficient of variation of Y = ay Illy. and Holand, I.: Risk Assessment of Offshore Structures - Experience and Principles. This is briefly referred to in section 13.4. Tables 13.1 and 13.2 show only two of many possible ways of classifying gross errors. t 62 3. a Assuming that the true values are known (i.e. from a level 2 analysis) and the variables ranked (taking due account of sign) so that X are (11.31) (11.32) where n R + ns = n, the total number of basic variables, the quantities R1 and Sl may be termed the leading resisting and loading variables, respectively. This procedure is illustrated in example 6.4. Example 6.4. Consider the simply supported beam shown on figure 6.3. The beam is loaded with two concentrated loads Xl and X 2 with the means E[X I] = E[X 2] = Il and the covariance matrix c-X = where -1 < p < 1. 206113020 - 543210 PREFACE Structural reliability theory is concerned with the rational treatment of uncertainties in structural engineering and with the methods for assessing the safety and serviceability of civil engineering and other structures. FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY corresponds to failure. The one-one relation between P f and {3 is given by equation (5.33) (5.33) When the basic variables are non-normally distributed this one-to-one relation is not valid. BIBLIOGRAPHY [6.1] Ditlevsen, 0.: Fundamentals of Second Moment Structural Reliability Theory. In writing this book we have tried to bring together under one cover the major components of structural reliability theory with the aim of making it possible for a newcomer to see and VI study the subject as a whole. Start Iteration No. 1 2 3 4 111 40'10- 7 116'10-7 158.10- 7 167'10-7 167.10- 7 cxl 0.58 0.35 0.23 0.23 0.23 cx2 -0.58 -0.90 -0.96 -0.97 -0.97 cx3 -0.58 -0.25 -0.14 -0.11 -0.11 Table 5.2 The mean value for the moment of inertia corresponding to ~ = 3 is 111 = 167 '10- 7 m4. Systems: In general, the aim of design is to provide an assembly of components which when acting together will perform satisfactorily for some nominal design life either with or without maintenance. They cannot be classed as improvements unless the new procedures result in improved standards of safety and/or reduced costs of construction and maintenance. This is illustrated by the following example. 178 11. Most computer routines use the sum of 12 or more independent rectangularly distributed random numbers rio If the latter are generated in the interval [0, al, their sum ~ can easily be shown to be approximately normally distributed with mean J.l t given by J.l t = an/2 (A.ll) and variance a~ by (A.12) For the simple case when a = 1 and n = 12, f 12 = L: r i -6 i=1 ~' given by (A.13) 253 APPENDIX A. The corresponding density function fx is also illustrated in figure 3.13. Cumulative frequency diagram for yield stress of mild steel plates. , zn) = 0 (11.19) The reliability index {3 is defined in Z space as the shortest distance from the origin to the failure surface and is given by (see (5.34)) n {3 = min zEaw 1 (,L; z~)2 i=l (11.20) 189 11.4. METHODS FOR THE EVALUATION OF PARTIAL COEFFICIENTS The point on the failure surface which is closest to the origin is referred to as the design point (see figure 5.5) and has co-ordinates ({3O:1 ' (3O:2' •.. 68 4. A.: Mean Upcrossing Rates for Sums of PulseType Stochastic Load Processes. 0.5 l.0 247 BIBLIOGRAPHY Let us now undertake an unconstrained minimisation of E[C T (g)] with respect to the quantity ilR and denote the minimum value of ilR by ilR,opt(g). Combinations of time-varying loads: When a structure has to resist a number of stochastically independent time-varying loads, it is clear that the probability of two or more loads exceeding their characteristic values simultaneously is small. Procedures such as this have already been used in the application of structural reliability theory to practical level 1 codes, e.g. [11.6], [11.10], [11.12]. = I Xi -IlX i a Xi ' i = 1, 2, ... & Wickham, A. [12.10) Cartwright, D. In evaluating the partial coefficients, the agreed policy was to achieve the same average reliability for components designed to the new code as the average inherent in designs to the previous code BS 153, but at the same time to reduce the scatter in the reliability of the various 197 1l.5. AN EXAMPLE OF PROBABILISTIC CODE CALIBRATION components. Neither of these facts is surprising since the code was originally based on deterministic concepts with no regard for the relative magnitude of the various uncertainties. If a tensile bar made of a brittle material fails due to a tensile force then such an element can reasonably be considered perfectly brittle, because its loading capacity is completely exhausted. 5. Substituting these values into equation (11.6) gives the required value of the remaining quantity generally a dimension. LKI, Heft 17, Technische Universitat MUnchen, 1977. If such uncoupling is possible, then the deterministic checking equation corresponding to equation (11.3) may be expressed as (11.6) where 'YR is a partial coefficient on the computed resistance 'Ys is a partial coefficient on the computed load effect and where the subscript d denotes the design value of the variable. In addition, there are significant differences in the average reliability of different types of component. The ratio, Cov[Xl' X 2] PX l X 2 = where aX l and (2.80) aX l aX 2 aX2 are the standard deviations of the random variables Xl and X 2 , is called the correlation coefficient. For information of these distributions the reader should consult a standard text, e.g. (3.5]. FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY With some types of component, large quantities of data may be available on time to failure and a distribution may be selected which gives the best fit to the data. A system of single elements is a series system if it is in a state of failure whenever any of its elements fails. Example 5.1. Consider the fundamental case with only two basic variables (a load variable S and a strength parameter R) and a failure function f 1 : R 2r>\ R, where fl (r, s) = r - s (5.5) The failure surface, the failure region and the safe region are shown in figure 5.2. The corresponding safety margin Ml is given by (5.6) An equivalent failure function is f 2 : R2~R, r f 2 (r, s) = Qns = Qnr - Qns where (5.7) with the safety margin M =Qn!!:=QnR-QnS 2 S (5.8) failure surface failure region safe region Figure 5.2 5.3 RELIABILITY INDEX FOR LINEAR FAILURE FUNCTIONS AND NORMAL BASIC VARIABLES For the case of a linear safety margin M and normal basic variables, the reliability index fJ is defined by (5.9) where J.lM is the mean of M and aM is the standard deviation of M. The English Universities Press, 1963. A.: Probability, Statistics and Decision for Civil Engineers. For example, the actual characteristic value of the 28-day cube or cylinder strength of concrete is likely to depend on the particular supplier or contractor and is not known in advance. 67 Chapter 4 FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY 4.1 INTRODUCTION Structural reliability analysis differs in many important ways from reliability analysis as prac· tised in, for example, the electronics and aero-space industries, in spite of the fact that the underlying probabilistic nature of the problems are the same. A point estimate is a single estimate of the parameter whereas interval estimates allow certain additional confidence or probability statements to be made. The term safety element is used as a generic term for partial coefficients and additive safety elements (see section 11.3.3). These variables called basic variables can be geometrical quantities (e.g. the area of the cross-section of a beam), material strength (e.g. rupture or yield strength) and external loads (e.g. traffic loads, wave or wind loads). [12.2] Baker, M. Part of the problem, therefore, is to predict the magnitude of these extreme events. In these cases it is necessary to compute the reliability function and hazard function for the system from a knowledge of the reliability functions of the components and a knowledge of how the components are inter-connected.·In general, account should also be taken of the possibility of detecting and replacing components which have failed. Let the starting seed io = 1234567 « 2 26). 168), (11.16) In principle, if there are n time-varying loads, it is necessary to undertake n design checks (equation (11.6» on the structure, using a separate set of ljio factors for each check and with lji Ojj = 1 for the jth check. Figure 12.11 shows the elevations of a structure intended for barge launching at a location in the North Sea with a mean water depth of 156 m. In general, the integer constants a and m are chosen to obtain the longest possible cycle. This is also a pre-requisite for the procedure described in section 11.4.2. The choice is generally made by a process of probabilistic calibration to an existing code. No estimator, however, has all these properties and in practice the choice of estimator is governed by the particular requirements of the problem, or expediency. This is because of the discrete nature of many structural components (e.g. rolled steel beams) and the need to round up to the next section size above when designing. is the mean of Xi' and 1 Vi is the coefficient of variation of Xi' Again, if the parameters!lx and V· are known, and x and ~ are given, then 'Y' can be i I sPi J evaluated from a knowledge of the sensitivity factor a i . It should be noted that F X n (x) may also be interpreted as the probability of the non-occur- > x) in any of n independent trial~,so that equation (3.5) follows iIjlme\ I \ I , 9l diately frlom the multiplication rule for probabilities. This can easily be demonstrated by considering the fund&mental case treated in example 5.2. In the fundamental case only two uncorrelated basic variables Rand S are involved. 3.3.1 Type I extreme-value distributions (Gumbel distributions) Type I asymptotic distribution of the largest extreme: If the upper tail of the parent distribution falls off in an exponential manner, i.e. Fx(x) = 1-e- g (x) (3.9) where g is an increasing function of x, then the distribution function Fy of the largest value Y, from a large sample selected at random from the parent population, will be of the form Fy(Y) = exp(-exp(-a(y -u))) a>O (3.10) Formally, Fy will asymptotically approach the distribution given by the right hand side of equation (3.10) as n -+ 00. See also chapter 7. However, the ratio kIn is a statistical variable whose sampling distribution, and in particular variance, depends on the number of trials n. As discussed in chapter 1, the calculated reliability or failure probability for a particular structure is not a unique property of that structure but a function of the reliability analyst's lack of knowledge of the properties of the structure and the uncertain nature of the loading to which it will be subjected in the future. , xn). SPECTRAL ANALYSIS OF WAVE FORCES In order to make equation (B.4) linear, eis minimised in the mean square sense and then dropped (see [B.3]). The only other possibility is Monte-Carlo simulation, but this too has its limitations. Deep-Sea Oil Production Structures. In this chapter it will be shown how this can be undertaken. The most advanced methods are the level 3 methods. omission of heat treatment - Incorrect construction · .. If bars are supplied by a number of different manufacturers, systematic differences between manufacturers will be evident even for nominally identical products (e.g. 20 mm diameter bars) because of differences in rolling procedures. 13.3 INTERACTION OF RELIABILITY THEORY AND QUALITY ASSURANCE 243 % of total 60 A. [13.6] Ditlevsen, 0.: Formal and Real Structural Safety. 4. The quantity I WjO j j given in the last row of the table is the ratio of the amount of steel used when .P 199 11.5. AN EXAMPLE OF PROBABILISTIC CODE CALIBRATION ft = 0 .63 X 10-2)2 (12.79) i=l Equations (12.73) and (12.78) together with equation (12.79) provide the basis for an efficient iterative method for calculating (3. & Singpurwalla, N. This should not, however, be seen as a limitation, since the aim is not to produce a perf.?ct image of reality (an impossible task), but to develop a mathematical model of the phenomenon which embodies its salient features and which can be used to make optimal design decisions using the data available. For this reason (see page 53) such loads are well represented by normal probability distributions. Some further results are given in [13.1]. A typical example of perfectly ductile behaviour is shown in figure 7 .2, where it is assumed that the load p can be maintained during an increasing displacement. This may be interpreted physically as the distribution that would be obtained if the maximum lifetime load were measured in an infinite set of nominally identical structures. A gross error should not therefore be considered as some extreme value in the tail of the probability distribution used to model a particular random variable, but a discrete event G which radically alters the probability of failure by changing the models that are applicable. This is shown in figure 13.2 for the set of parameters given in table 13.4. An important result for this particular set of assumptions is that if the quality assurance process is able to restrict the frequency of gross errors to less than 2%, then ilR,opt is very insensitive to the occurrence of gross errors of any magnitude. UNIFORM RANDOM NUMBER GENERATORS Most digital random number generators are based on uniform pseudo-random number generators of the multiplicative congruence type. The question that must be asked is whether the model is suitable for the particular application where it is to be used. 23, 1952. V.: Statistical Methods in Structural Mechanics. For the jth design check equation (11.16) may then be re-written as (11.17) where gd = (gdl' ... The probability of failure Pf is then defined as Pf = P(S;;;' R) (7.1) assuming that the failure condition is R - S';;;; O. A clear understanding of these differences is helpful and they will be discussed later. In other words, it is necessary to synthesise probabilistic models for the two parts of the problem, including, on the one hand, all the uncertainties affecting the loading, and, on the other, all the uncertainties affecting the strength or resistance of the structures. It is sometimes argued that the normal distribution should not be used to model resistance varia- bles because it gives a finite probability of negative strengths. The upper bound is especially useful so its derivation will be shown here. Annals of Mathematical Statistics, Vol. and Hibbard, H. Variations due to waves in the surface elevation of the sea X(t) at any fixed point remote from the shore can be shown to have a first-order distribution Fx which approximates very closely to the normal distribution (for periods of time in which the sea-state can be assumed to be stationary). BIBLIOGRAPHY [13.1] Baker, M. 170 180 190 200 210 46 3. Clearly, there is only one source of loading, but the way in which it is classified and modelled is dictated by the failure mode being analysed. Results of an Enquiry. E.: A Statistical Theory for Hydrodynamic Forces on Objects. For the same reason it is appropriate that the characteristic value gk of each permanent load G is taken as its mean value!la' !la may be considered to be the average permanent load taken over all nominally similar structures and obtained by using mean dimensions and mean densities. 2, July 1968. [3.8] Gumbel, E. The extent to which these results can be generalised depends on circumstances, but it is considered that under many conditions the optimisation of expenditure on the control of gross errors can be undertaken independently of the choice of partial coefficients. 31-44. A gross error is defined as a major or fundamental mistake in some aspect of the processes of planning, design, analysis, construction, use or maintenance of a structure that has the potential for causing failure. At the worst, the total expected cost differs by only 15%. Institute of Building Technology and Structural Engineering Aalborg University Centre Aalborg, Denmark MICHAEL J. However, readers who have had no training in this branch of mathematics would be well advised to study a more general text in addition. / V Mill »Y. It is assumed in the following that the distribution function is known or has been postulated and that its parameters are now to be estimated. Partial coefficients are also essential for the rational treatment of load combinations, and in particular for situations in which the total load effect in part of a structure is the difference of two load effects of approximately similar magnitude but originating from different load sources - e.g. the effects of gravity loads and wind loads in the up-wind columns of a tall building. SPECIAL CASES: GENERATION OF RANDOM DEVIATES HAVING NORMAL AND LOG·NORMAL DISTRIBUTIONS The normal and log-normal distributions are two of the Class B functions, but because of their frequent use they deserve further attention. ,xn)dx l dX 2 ... BIBLIOGRAPHY 111 [6.3] Hasofer, A. [12.35] Sarpkaya, T.: The Hydrodynamic Resistance of Roughened Cylinders in Harmonic Flow. The latter might be the specified design life or any other period of time. Example 13.1. Assume, for the sake of simplicity, that a structure has a resistance R which is dependent on only one basic variable, the yield stress of steel, and that it is subjected to a single load effect S. Negligence H. Before this question can be answered it is necessary to define exactly what the variable X represents. Let f(X) = f(X I , ... and Amin, M.: Safety Factors and Probability in Structural Design. [12.3] Baker, M. For example, if the particular variable of interest is the maximum of many similar but independent events (e.g. the annual maximum mean-hourly wind speed at a particular site) there are generally good theoretical grounds for expecting the variable to have a distribution function which is very close to one of the asymptotic extreme value distributions. 73-98. In practical codes the design requirements may be made considerably simpler. This difficulty cannot be escaped, but the probability of large errors occurring decreases as the sample size increases. Note that (2.83) Therefore, the mutual correlation ~tween by the so-called covariance matrix C defined by random variables Xl' X 2 , .•. In particular, it concerns: the number of partial coefficients or other safety elements to be used, their positions in the design equations, and rules for load combinations. Journal of Applied Physics, Vol. 13.2 GROSS ERRORS 13.2.1 General In chapters 3 and 12 a considerable amount of space is used to discuss the probabilistic modelling of loads and resistance variables. They should be thought of as intermediate results in a decision-making process and with little or no absolute meaning of their own. Finally, the reliability index can be defined in the z-coordinate system as shown above. M. and Schneider, J.: Untersuchungen zur Struktur des Sicherheitsproblems bei Bauwerken. R., Schafer, R. On each step of the iteration new values for a~. 53 3.4 MODELLING OF RESISTANCE VARIABLES - MODEL SELECTION The normal (Gaussian) distribution: As discussed in chapter 2, this is one of the most important probability distributions. Choice of reliability function: In many areas of electronic, electrical and mechanical engineering, data are available for the time to failure of item& such as relays, bearings, driveshafts and many other components. 4.3.5 The General Case Only in the simplest of structures can the reliability be expressed in terms of the two random variables Rand S. Shubert: Probabilistic Models in Engineering Sciences. 236 [12.21] 12. The nominal loads specified in most loading codes vary rather widely in terms of their probability of exceedance. However, the form given in equation (3.57) is in fact preferable from a computational point of view. +------------.-\,---------=::::::=::======= ~ c1 ; 786.5 I I I I I "-initial cost for IlR ; 78.65 500 +_--r---+---~-+_--r-_+----+_--1----r-~---+_--~--~g 0.1 Figure 13.3 (p(g); 0.05). & Woodgate, R. 1 n ='j1 L;xi i=l (3.55) and &2 1 n "x~1 n":::;" i=l =- - P' 2 (3.56) giving 1 n n i=l i=l 2 &2 =-n(L;Xf -(L;x i) In) (3.57) Alternatively &2 may be expressed as 1 n ~ (3.58) n":::;" 1 i=l where p. , x n) is a realizax is a point in an n-di- tion of the random vector X = (Xl' ... and Overvik, T.:Parameterization of Wave Spectra and Long Term Joint Distribution of Wave Height and Period. ---------- ~--------~~------~-Xl / Figure 5.1. 83 5.3 LINEAR FAILURE FUNCTIONS AND NORMAL BASIC VARIABLES Let f be a failure function. What is the optimal period of time that should elapse before a system or component is inspected or replaced? PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES The maximum-likelihood estimators e of the parameters e are defined as the values of e tha} maximize L, or, equivalently and more conveniently, the logarithm of L. and &2 given by equations (3.56) and (3.57) are thus the moment estimators of J.L and a 2, respectively. For situations in which only one set of data and no other information is avaiable, the approach is straightforward. Therefore, (10.7) where Vx (0' T ~ 1. If, however, data from a number of manufacturers are combined, the distribution becomes highly skew. Reliability (Engineering) 1. The likelihood function of this sample is defined as n L(Olx l ,x 2 , .. 27 0 .25 = 0.3051 2 (kN) m 6.2 CONCEPT OF CORRELATION 99 Consider again the set of n random variables X = (Xl' ... and Wootton, L. Assuming that the limit state function may be split into a resistance term R and a load effect term S, as in equation (11.6), it has been proposed [11.5] that the sensitivity factors should be expressed as Cl: R . It should be noted, however, that the precise form and parameters of the extreme-value distribution depend very strongly on the autocorrelation function of the loading process X(t). D. Yield stress (N/mm2) 500 480 dynamic yield stress 0Yd 460 static yield stress a ys 440 420 15 10 5 25 20 30 length (km) +---_.--~.---_.~--,_--~----,_~~----._L-_,----1_--~ o 5 10 15 20 25 30 35 40 45 50 bar number Figure 3.8. Within-cast variations in the yield stress of a 20 mm diameter hot-rolled reinforcing bar. Lectures on Structural Reliability (ed. Hence iHT(t) = r r fT(T)dT = Xe-ATdT = e- At (4.11) and thus (4.12) For this particular form of distribution of time to failure, the hazard function is a constant, and thus the probability of failure per unit time is independent of preceding events. The different techniques of parameter estimation summarised below correspond to the use of different functions of the sample data and give rise to different estimators for the parameters. A small desk computer or pocket computer is suitable for this purpose. Yield stress (N /mm 2) 550 500 450 Bar diameter (mm) 400+-____~------~------+_----_+--~~ 10 20 30 40 o x Baker and Wickham (1979) o Baker (1970) • Bannister (1968) Figure 3.9. Mean yield stress for hot· rolled high yield bars of different diameters. Recalculate the probability under the assumption that the various R j are mutually fully correlated (p = + 1). JYES L -1 - - - - - - - -~- - - - - - - Design components to BS 5400: Part 3 (r) t Calculate P = .2 Wi Pfier) i 1 t Determine S = .2 Wi Ll (P fi er), Pft) i STOP -- -, I I I t Determine failure probabilities Pfi - r I I I I I I I I I I I I --------------~------------~ Figure 11.4. Probabilistic calibration of BS 5400: Part 3 to BS 153. We now return to the question of control measures. 1979 (unpublished). S.: The Statistical Distribution of the Maxima of a Random Function. It is natural, therefore, to think in terms of the life of electrical and mechanical systems, although the precise time at which))failure)) occurs may be difficult to establish since the definition of failure may be fuzzy or somewhat arbitrary. Let (B.9) Pre-multiplication by ~T and substitution in equation (B.8) gives (B.10) or (B.ll) The generalised mass matrix M and stiffness matrix K will be diagonal due to orthogonality but the damping matrix C may be full. and B. It is assumed that the functions fi' i tions so that inverse relations exist = 1,2, ... 95 Chapter 6 EXTENDED LEVEL 2 METHODS 6.1 INTRODUCTION In chapter 5 a detailed presentation of level 2 methods was given. Exercise 3.3. Given that the column discussed in example 3.3 is subjected to an axial load of 1500 kN, calculate the probability that this load exceeds the load-carrying capacity. In most practical situations the form of the failure function g will be such that it is not explicitly differentiable. f (x)=--exp(--(X a-.(2ir 2 a 2 » (3.50) Assume that a random sample of n observations of X has been obtained, (xl' x 2 ' ... 103, No. ST5, May 1977, pp. [12.22] Laboratorio Nacional de Engenharia Civil: Wind in Western Europe. Starting with an integer »seed» io the first pseudo-random number r 1 in the interval [0, IJ is obtained from ai m ~=j 1 + ai -j m i 0 1 =j +-1. Holand, 1. This is mainly because Rand S are not known or are not convenient mathematical expressions, e.g. R = function(material properties, dimensions) S = function(applied loads, densities, dimensions) (4.37) Indeed Rand S may not be statistically independent for example, cross-sectional dimensions affect both sectional strengths and dead loads. To distinguish these two types of element behaviour the symbols shown in figure 7.3 can be used. [4.13] Pugsley, A. When more than one time-varying load variable acts in combination on a structure then the above simplification cannot be used because determination of the distribution of the combined load effect requires knowledge of the detailed variation with time of the individual loading processes. For most practical problems neither task is easy since there may be a number of distributions which appear to fit the available data equally well. API RP 2A Seventh Edition, Jan. Such control will, in general, reduce the probability of structural failure and thus increase safety. of Building Technology, Lund Institute of Technology, Report 60, 1975, Lund, Sweden. and Shinozuka, M.: The Analysis of Structural Safety. Readers unfamiliar with the various concepts should also study a specialist text [3.111, [3.51, [3.81. -p.») X ~ _1. The parameters of the probability distributions of the important basic variables are given in table 12.1. Using this data, it was found that failure of the member occurs with a probability of about 3.0 X 10-7 in a reference period of 25 years, but no particular significance should be attached to this number. G.: Note on the Distribution of the Largest Value of a Random Function with Application to Gust Loading. ·,xn)= flfx(xiIO) i=l (3.59) L expresses the relative likelihood of having observed the sample as a function of the parameters Referring to equation (2.68) it can be seen that the right hand side of equation (3.59) is the e. Cross-sect jon of rejnforced concrete column. 11.3.1 Limit state functions and checking equations As discussed in chapters 4 and 5, the general conditions for a limit state not to be exceeded may be expressed as (11.1) where X are the n basic random variables which influence the limit state, and f is the limit state function (failure function), The variables X may be sub·divided into variable loads and actions Q, permanent loads G, material properties E, geometrical parameters D, and model uncertainties Xm (see equation (1.1)). We should like to acknowledge the major contributions to the field of structural reliability theory that have been made by a relatively small number of people, mainly during the last 10 to 15 years, and without which this book would not have been possible. What is the probability that the process (10.1) has a value larger than x(t) = ~ during the reference period 0 ..;; t ..;; T?), = r(·), (11.5) 182 11. The latter is not always easy to verify because existing codes may not have been in use for a sufficiently long period of time and structures may have been subjected to only a fraction of their design loads. Cornell: Probability, Statistics and Decision for Civil Engi- neers. [6.5] Rationalisation of Safety and Serviceability Factors in Structural Codes. ,Ok' From equation (2.35) the jth moment of X is given by °° (3.46) °° Since fx is a function of the k parameters l' 2 ' ... In the preceding example, this was the 50 year extreme maximum distribution. 1079 ·1093. Method of moments: Let the variable of interest X have a probability density function fx' with parameters 1 , 2 , ... Deutsches Hydrographisches Zeitschrift, Hamburg, Reihe A (8°) Nr. 12. The analysis of many structural failures (see e.g. [13.2]) shows that the majority could not have been prevented by minor increases in partial coefficients. [12.12) [12.13) Davenport, A. It follows from (6.1) that if a large and positive value of Cov[X I , X 2] occurs then the values of Xl and X 2 tend to be simultaneously large or small relative to their means. J.: The Reliability of Reinforced Concrete Floor Slabs in Office Buildings - A Probabilistic Study. For this reason, some results that were obtained during the probabilistic calibration [11.6] of the U.K. Steel Bridge Code BS 5400: Part 3 [11.3] are included here as an illustration of the method. is a factor which depends on the relative importance of the ith resistance variable, and as.,I is a factor which depends on the relative importance of the ith loading variable. ,w m) is a set of weighting factors indicating the relative importance of each of the m structural components included in the partial factor evaluation. What is the probability that an additional layer of asphalt will be placed on the bridge without removal of the original surfacing and how should this be allowed for? , xi, ... Such changes generally increase design costs and increase the risk of major errors being made. Such tests are widely described, e.g. [3.5], and will not be given here. Determine the mean and the variance for {X(t)} and for rX(t)}. ACI Monograph No.1, 1964. 3rd International Conference of Structural Safety and Reliability, Trondheim, 198!. Uncorrelated variables YI and Y2 are according to equation (6.11) determined by Y=ATX Bx. where the transformation matrix is_determined by the eigenvectors for The eigenvalues AI' A2 and eigenvectors vI' v2 for' Cx are easily determined in this simple case Vl = 4 (1,1) v2 = Yf (1, -1) The transformation AT =Yf [1 A is therefore given by 1] 1 -1 and, according to the equations (6.11) - (6.13), = =T= = Cy=A CxA ~ 103 6.3 CORRELATED BASIC VARIABLES ! f2 A B Xl 4 a ~ ~' a AI/ £ a J1v Figure 6.3 Un correlated and normalized variables Zl and Z2 are finally determined by _ Y l - E[Yl] _ Zl a - Y1 v'2 2 (Xl + X 2 - 2J1)/av'I+P (6.16) (6.17) From the matrix equations (6 .11) - (6.13) it is seen that 1 Z = Cy - Z = (A?C:x A) -2 AT (X - E[X]) 2 (V - E[Y]) or 1 (6.18) Note that in the z-coordinate system we have 1 1 zT z = (x - E[X])TA(ATCxA)- 2 (ATCxA)- 2AT (x - E[X]) = (x - E[X])TA(ATCxA) -1 AT(x - E[X]) = (x - E[X])TCi (x - E[X]) (6 .19) The definition of the reliability index ~ by Hasofer and Lind can now be extended to (6 .20) where z is the failure surface in the z-coordinate system and coordinate system . Example 3.6. Consider again the reinforced concrete column discussed in example 3.5. Since the strength of the concrete is assumed known and the strengths of the reinforcing bars have been assumed to be independent, it may be concluded that the load-carrying capacity of the column R is normally distributed. The book should be of value to those with no prior knowledge of reliability theory, but it should also be of interest to those engineers involved in the development of structural and loading codes and to those concerned with the safety assessment of complex structures. The Gumbel density function fp and the corresponding normal density function np are shown in figure 6.6. According to the equations (6.35) and (6.36) np and the corresponding distribution function Np are connected with fp and Fp in the following way (6.42) (6.43) where x; is the third coordinate of the design point Exercise 6.5. Consider again the elastic beam treated in example 5.6, but assume now that the load P is Gumbel distributed with the mean I1p = 4 kN and the standard deviation up = 1 kN. Report DMCA Structural reliability theory is concerned with the rational treatment of uncertainties in struc tural engineering and with the methods for assessing the safety and serviceability of civil en gineering and other structures. This approach is the only one possible when the number of trials is small. A.: Bayesian Statistical Decision Theory and Reliability-Based Design. " - exp[-- "(x. [3.11] Mann, N. To overcome the problem that the definition space for the set of basic variables Xl' ... APPLICATIONS TO STRUCTURAL CODES 11.4.3 General method for the evaluation of partial coefficients Practical codes should have the smallest number of partial coefficients that is consistent with reasonably uniform standards of reliability; moreover, the same partial coefficients should be applicable to a wide range of structural components. Example 3.7. Many friction problems are governed by relationships of the form P = kei.lC< where k, J.1.and (3.43) C! are variables. The uploader already confirmed that they had the permission to publish it. In this case, the values gd and qd to be used in the design or safety checking process (cf. Such lumping of data is often necessary when the sample sizes would otherwise be very small, but this should be avoided if possible. Zn =]] Zj j=1 (3.41) Clearly Y given by n Y = QnX = QnZ 1 + QnZ 2 + ... It was designed in accordance with the rules of the American Petroleum Institute [12.1] with full allowance for dynamic response, and was used as a basis for a sensitivity study [12.23], [12.2]. It is also clear that the probability density function fx representing all bars, irrespective of size or manufacturer, will not be of a simple or standard form (e.g. normal, lognormal, etc.). From equations (4.34) and (4.35) E[QnP) = £n/lp -t a~ = £n(250) - 0.5 X (0.0606) = 5.49115 77 4.3.4 TREATMENT OF A SINGLE TIME·VARYING LOAD Similarly, E[Qr.C] = - 0.1103, E[QnE] = 5.3227 and E[QnL] = 8.2829. Then ~ _ 3125 X 1234567 _. Should the surfacing be treated as a permanent or a variable load? If the probability density function of the random variable X, for known parameters 0 is written as fX (x I0) then the predictive density hx for uncertain "0 is given by hX(x) = Lfx(xIO)f~ o (3.63) (0 Iz)dO where f~ (0 Iz) is the posterior probability density for 0 given a set of data z f'if (0 Iz) can be obtained from Bayes theorem (see equation (2.24» = (zl' z2' ... and Lamberson, L. 44 3. R.: The Estimation of Extreme Winds. 258 APPENDIX B. (y2 _ y2) 2 1 2 so that the failure criterion (6.32) can be written (6.33) where y land y 2 are realizations of un correlated random variables Y 1 and Y 2' The reliability index {3 can now be calculated by the iterative method shown in Chapter 5. 3.2.1 Derivation of the cumulative distribution of the ith smallest value of n identically distributed independent random variables Xi Assume the existence of a random variable X (e.g. the maximum mean-hourly wind speed in consecutive yearly periods) having a cumulative distribution function Fx and a corresponding probability density function f x ' This is often referred to as the parent distribution. , Yn) and (2.74) is the Jacobian determinant. Generation of random normal deviates from the sum of n rectangularly distributed random deviates. (x. x I ...,-:::: ~ rt" •. Annals of Math. Baker, Michael J. M.: Waves at Ocean Weather Ship Stiltion India. Thoft-Christensen), Aalborg University Centre, Aalborg, Denmark, 1980, pp. Y. BIBLIOGRAPHY [12.4) 235 Battjes, J. The problem of modelling is completely different. Lack of experience 40 D. Practical rules can be established giving the necessary number of trials for any given magnitude of Pf' The second approach is to fit an appropriate probability distribution f M" to the trial values of M' , using its sample moments. Let FR. A logical sequence of steps is as follows • set limits on the range of structures and materials for which the code will be applica::'le, • specify the deterministic functional relationships to be used as the basis for each design clause, • select the general form of the probabilistic models for the various load and resistance variables and model uncertainties, 180 11. Taking into account the consequences of system malfunctions, is it economic to increase the expected life by providing more component redundancy? , x n) = 0 (11.18) to a failure surface in standard normal space f(zl ' z2' ... Class B: The distribution function Fx has an inverse Fi1 which cannot be expressed in closed form In this case the general procedure is the same, but the inverse function has to be evaluated either graphically, by numerical integration, by table look-up and interpolation, or by fitting an appropriate polynomial. Report No. W 278, Aug. 84 5. 194 1l. The two random variables Xl and X 2 are said to be independent if (2.67) which implies (2.68) By integrating (2.66) with respect to xl one gets the conditional distribution function _[l~ f X (x~ , x 2)dx~ (x) X2 2 lo X2 FXIIX2(xllx2) - f (2.69) Next by integrating with respect to x 2 the so-called total probability theorem is shown (2.70) Example 2.14. Equation (11.6) is the most general form of the checking equation for a structure in which Rand S can be uncoupled. Exercise 2.10. Such a system is also called a weakest-link system. 14, 1957. Should variations in density as well as variations in thickness be modelled? In such cases, these »loads» are strictly resistance variables from a reliability viewpoint. Reliability, ASCE, Tucson, Arizona, January 1979. The maximum deflection can therefore be written where k is a constant. If the histogram is clearly bi-modal when a uni-modal distribution is being fitted to the data or if the sample appears to be truncated when the variable is assumed to be unbounded, checks on the data source are clearly required. The advantage of this method is that it is accurate over the complete range and depends only on the randomness and independence of r 1 and r 2 . The coefficients are chosen to give unbiased and highly efficient estimates for samples of particular size. This is not the case in the next example, where the failure function is non-linear in the two basic random variables which are also correlated. Influence of Gross Errors. The responsibility for this book must, however, rest with the authors and we should be pleased to receive notification of corrections or omissions of any nature. , n (6.15) 102 6. + c a,11.. Thus the value of the diameter which will result in failure 50% of the time is 80.83 mm. (11.29) '" asa s ,1. A.: A Probability-Based Structural Code. neglect of lateral torsional buckling concept - Incorrect nature of use assumed · .. These are called asymptotic extreme-value distributions and are of three main types, I, I! and II!. Manually recorded or copied data have a high probability of containing at least some transcription errors. For the specific realisation shown here, the instants of time t I , t2 and t3 for maximum of PI (t), P2(t) and PI (t) + P2(t) are all different. [13.9] Joint Committee on Structural Safety, CEB - CECM - CIB - FIP - IABSE - lASS RILEM: General Principles on Quality Assurance for Structures. A number of limiting cases will now be studied. E.: An Introduction to Random Vibrations and Spectral Analysis. Let Q B and MB be the shear force and bendmg moment at the support B. [13.18] Tveit, O. McGraw-Hill, N. APPLICATIONS TO FIXED OFFSHORE STRUCTURES Kinsman, B.: Wind Waves. (Answer: P(~ > 5) = 0.075). Hence, the process of designing a structural member involves • • • • determination of the design loads Ci d , selection of materials and determination of the design values of their relevant mechanical properties ed' selection of primary dimensions dd to satisfy the particular engineering and architectural requirements, and determination of the remaining unknown dd to satisfy equation (11.4). +(1) n-i n J (3.4) Exercise 3.1. Show that equation (3.4) can be derived from equation (3.3) by expanding (1 - Fx(x»n-i and integrating by parts. It should be noted that this is an entirely deterministic concept. 37 Chapter 3 PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES 3.1 INTRODUCTION In this chapter the aim is to examine the way in which suitable probabilistic models can be developed to represent the uncertainties that exist in typical basic variables. The next step is to estimate the parameters of the selected distribution using one or more of the techniques described in section 3.6.1 below. If the loading process X(t) can be assumed to be ergodic (see chapter 9), the distribution of the largest extreme load can be thought of as being generated by sampling the values of x max from successive reference periods T. , n (11.21) with (11.22) By using the inverse mapping x* =F}l ((z*)) i 1 i = 1, 2, ... [12.7) Brebbia, C. 11.4 METHODS FOR THE EVALUATION OF PARTIAL COEFFICIENTS Any reader who is unfamiliar with the theory of level 1 codes may be somewhat concerned by the apparent complexity of the safety checking rules set out in section 11.3 and by the apparent arbitrariness of some of the steps. If the distribution parameters have been estimated graphically, this step will have been taken as part of the estimation procedure. The general procedure is to obtain estimates of these unknown parameters in terms of appropriate functions of the sample values. It is important to note that the idealisation of a structure by a series system as in figure 7.5 is only related to the failure interaction. error in computer program - Misinterpretation of units · .. This leg member acts as a strut carrying a combination of axial load and moment, and failure was deemed to occur at the collapse load predicted by the API design rules [12.1], but treating the dimensions and material properties as random variables and setting the permissible stress equal to the yield stress. The safety and serviceability of a structure are influenced as much,if not more, by the nature of the control measures that are in operation as by the magnitude of the partial coefficients that are used in design. i1 = (aio - j1 m) is the seed for the second random number. Specialty Conf. In this case, the safety or serviceability of a structure (the probability that the limit state defined by the particular form of the functions r and s will not be reached) can clearly be increased or decreased by adjusting any or all of the (n - 1) independent design values xd (e.g. Cid or e d) and the two partial coefficients 'YR and 'Y s. Chapter 11, on the application of reliability theory to the development of level 1 codes, attempts to address many of the practical problems faced by code writers in the selection of partial coefficients (partial factors); and in chapter 3, the modelling of load and resistance variables has been approached with applications strongly in mind. Determine PM F so that!3 = 2.9 with the same failure function and with uM F = 0.08PMF· BIBLIOGRAPHY [5.1] Cornell, C. ,n from equation (12.73). , x~, ... The terms on the left hand side are of the standard form for a linear multi-degree of freedom system with viscous damping, except for the error term e which involves a quadratic term in i: . (For a precise definition of these terms, see for example, [3.11]). [12.29] Moses, F.: Reliability Analysis Format for Offshore Structures. EXTENDED LEVEL 2 METHODS 108 Exercise 6.4. Determine the reliability index {3 for the safety problem formulated in example 6.7. 6.4 NON -NORMAL BASIC VARIABLES Until now only second order information has been taken into account when evaluating the reliability of a structure. These predictions may then be used to answer such questions as: What is the probability that the actual life of a particular system will exceed the required or specified design life? This will be recognised as the normal approach to design. = The quantity S is given in the penultimate row of table 11.4 for each of the sets of partial coefficients calculated and can be seen to increase as additional constraints are introduced. 2nd International Symposium, Glasgow,

July 1981. K. In addition to the general method described above, a number of special methods exist for normal variables. [12.23] L. 2 2 Note that equation (10.9) is a generalization of the well-known convolution integral in elementary probability theory. For single distributions this requires just one set of data, but for the more complex mixed distribution models such as shown in equation (3.34) various sets of data are clearly required. [13.5] CIRIA: Rationalisation of Safety and Serviceability Factors in Structural Codes. In this case, however, it is the sample values xl' X2 ' •••, xn that are known and the parameters 0 that are treated as variables. Concrete, Vol. John Wiley and Sons, 1965. Solid Mechanics Division, University of Waterloo, Water· Canada, 1970. It should not be thought that the models are intended to be a perfect mirror image of reality, but rather as a »tool» in a decision making process, Depending on the nature of the decision to be taken, the »tool» may need to be changed. However, a thorough presentation of these problems is beyond the scope of this book. In this chapter, level 2 methods will be treated in a more detailed way and a number of simple applications will be shown. For such structures it is normal to specify an upper limit f on the imperfection magnitude, i.e. (11.9) In this case, f can be taken as the specified characteristic value of I, isp. As will be discussed later, there may often be advantages in using simplified design rules. , n and therefore, the set of variables x = (x l' ... The problem arises therefore of selecting the values of the parameters 8 for that distributicin. The effects of increasing r Q by, say, 20% will in general be • an increase in the nominal capacity of the structure to support the load Q, • an increase in the actual capacity of the structure to support the load Q, • an increase in the sizes of the structural members and the self-weight of the structure, • an increase in the cost of the structural system, • some increase in the actual capacity of the structure to resist any other load Q', and • an increase in the safety of the structure as characterised by a reduction in the probability that it will fail in any given reference period T. Substituting these values yields Var[Qn8) = 0.010013 and thus a~n0 = 0.10006. For a given structure each variable Xi' i = 1, ... APPLICATIONS TO STRUCTURAL CODES Assuming that the parameters !lXj and a Xj of the variable Xj are known or can be estimated, that x is given and that the reliability index ~ is specified, the evaluation of the SPj partial coefficient 'Yj requires only a knowledge of the sensitivity factor a j • Example 11.2. If Xi is a log-normally distributed resistance variable, then x '" _ _ _s_P,:-i-:::-::-::-::-c!lx. J = ae,(wt + = cf ,II·· Iip,t. Equation (3.4) give~ the probability distribution function for the ith smallest value of n values sampled at random from a variable X with a probability distribution Fx' Two special cases will now be considered in the following examples. It is a subject which has grown rapidly during the last decade and has evolved from being a topic for academic research to a set of well-developed or developing methodologies with a wide range of practical applications. Note that the variance Var[X i] = (5.2) at is equal to Cov[Xi' X). A histogram of some typical plate-panel imperfections (plate-panel out-of-flatness) obtained from measurements on the steel box-girder bridge at Aust in the U.K. is shown in figure 11.2. The quantity (/) is the ratio of measured imperfection to the specified maximum imperfection number 5 o 0.25 0.50 0.75 1.00 Figure 1l.2 Plate panel imperfections· Aust Bridge. It is normally denoted h and is given by h(t) = lim (4.9) lit .... The correct approach is to use the probability distribution for the load W corresponding to the maximum intensity in the reference period T for which the reliability is being determined. Berichte zur Sicherheitstheorie der Bauwerke, Lab. If X are non-normal then equation (11.24) is only approximate. PROBABILISTIC MODELS FOR LOADS AND RESISTANCE VARIABLES clearly has a relative frequency interpretation; i.e. if a very large sample of variable X is obtained at random, the proportion of values within the sample falling in the range xl < X .;; x 2 is likely to to be very close to P. [10.5] Nordic Committee on Building Regulations: Recommendation for Loading and Safety Regulations for Structural Design. Similar conclusions have been reached by others, e.g. [13.6]. 54 3. APPLICATIONS TO STRUCTURAL CODES It is therefore clear that the »best» codes are not necessarily those with the most scientifically advanced design clauses. If the aim is to design, as opposed to check, a particular structural member, it may often be pos- sible to invert equation (11.3) to give the minimum design value of some convenient resistance variable - for example, a dimension D' or a section modulus, e.g. (11.4) where f' is a function related to f, involving (n -1) quantities xd and m constants c. Example 7.1. Consider a simple structure consisting of two tensile bars as shown in figure 7 .6. Thus, if fH is an acceptable reliability for the structure, a satisfactory set of partial coefficients is given by (11.25) where x sPi is the specified value of the resistance variable X., and by I FX1((Z~))] where x SPj J (11.26) is the specified value of the loading variable X .. McGraw-Hill, N.Y. 1967. Hydraulic Engineering Laboratory Wave Research Projects. The reader is referred to the references at the end of the chapter. 11.5 AN EXAMPLE OF PROBABILISTIC CODE CALIBRATION The general method for the evaluation of partial coefficients which was described in the previous section involves a considerable amount of effort and computation and is not easily illustrated by a simple example. (ed.): Selected Papers on Noise and Stochastic Processes. I, 1950, Vol. The second step is to plot the data in the form of a histogram to check for outliers and to confirm that its shape does not deviate markedly from the shape of the density function being fitted. However, this apparent criticism can be assumed to be relatively unimportant if the strength of a component can be considered to be the sum of a number of independent random variables, thereby invoking the central limit theorem. 28, 1964. In this case, the best solution to the problem is to express each limit state equation or failure function in terms of the set of n basic variables X which affect the structural performance, such that 78 4. and J.1~. NKB-Report No. 36, Nov. If the design strength of a material is given b~' ed = e sp h m' where e sp is the sJJecified material strength and r m is a partial coefficient, an increase in r m will in general have the same effects as an increase in r Q . + cb ,11.. Exerci~e 3.2. Let Y be type II maxima distributed with distribution function Fy and coefficient of variationuy IIiy. Exercise ~2. [6.2] Ditlevsen, 0.: Uncertainty Modeling. G.: The Safety of Structures. The first few terms of this random sequence are: Term n o 1 2 3 in -"---- 1234567,,--- - 3281~627 "---"0.48901 9615183 ~0.14328 49781667 ~ 0.74185 0.27921 251 APPENDIX A. Setting (4.40) defines an (n - 1) dimensional hyper-surface in the n-dimensional basic variable space. Let Xl and X 2 be two random variables with the expected values E[X I] = JlX l and E[X 2] = Jlx 2 • The mixed central moment defined by (2.79) is called the couariance of Xl and X 2 . Solving (6.35) and (6.36) with regard to !l~. ASCE, EM6, Vol,109, Dec. The purpose of these is to illustrate the important principles and methods and to extend the scope of the main text with economy of space. G. Int. In t E [0; TJ), i.e. [0; T]) ... Finally, a note of caution. 3.7 INCLUSION OF STATISTICAL UNCERTAINTY As mentioned previously, the analyst is often faced with the problem of having insufficient data for one or all of the basic variables which affect the structural reliability. An appropriate choice for the target failure probability Pft for a new code is the weighted average of the failure probabilities exhibited by components designed to existing codes, provided that the least reliable component exhibited satisfactory performance in actual service. The formal use of goodness-of· fit tests in the context of structural reliability theory is therefore often limited. [13.8] Institution of Structural Engineers: Structural Failures in Buildings. McGraw-Hill, 1981. It arises as the limiting distribution of the largest value of many independent identically distributed random variables, when the parent distribution is limited to values greater than zero and has an infinite tail to the right of the form (3.20) 3.3.3 Type III extleme-vaiue distributions In this case only the type III asymptotic distribution of the smallest extreme will be considered. RELIABILITY OF STRUCTURAL SYSTEMS In this chapter the fundamental systems - series systems and parallel systems - will be defined and it will be shown how such systems can be analysed from a probabilistic point of view, both when no correlation between the elements exists and when all elements are equally correlated. I & 11,1979. [3.12] Mathieu, H.: ManuelllSecurite des Structuresll. [2.2) Benjamin, J. [3.5] Benjamin, J. This is illustrated for changes in standard deviation ax in figure 12.1a, and for changes in mean I1X in figure 12.1b. This is less justified for civil engineering systems, and the probabilities that are determined should not be thought of in this way. FUNDAMENTALS OF STRUCTURAL RELIABILITY THEORY under the condition that Rand S are statistically independent. This trans.formation is chosen in such a way that the values of the original density functions fXi and the original distribution function FXi for the random variables Xi are equal to the corresponding values of the density function and the distribution function for a normally distributed variable at the design point A (see figure 5.5), i.e. (6.35) (6.36) where the design point A = (x*l' ... Hence (B.7) where cf is a diagonal matrix of terms, referred to as fluid damping coefficients. A.: Methods at-Reliability Analysis for Jacket Platforms. Let the random variable Y be a function f of the random vector X = (Xl' ... 11.2 STRUCTURAL SAFETY AND LEVEL 1 CODES As mentioned in chapter 1, level 1 design methods were described as »design methods in which appropriate degrees of structural reliability are provided on a structural element basis (occasionally on a structural basis) by the use of a number of partial safety factors (partial coefficients) related to pre-defined characteristic or nominal values of the major structural and loading variables». The reason for this is that complete systems are more difficult and costly to test and secondly because it is often the aim of the design to provide much higher reliability for the complete system than for the individual components, by providing a high level of component and sub-system redundancy. we have 1 a , Xi 1 ip(-\ (Fx (x~))) = 1 fx(xn I (6.37) 1 (6.38) 109 6.4 NON-NORMAL BASIC VARIABLES Clearly the iterative method presented in chapter 5 for calculating the reliability index {3 for a given structure must be modified a little when the transformation shown above is used. 29-38. TA656.5.T47 1982 624.1 1 71 ISBN-13:978-3-642-68699-3(U.S.) 82-10277 ISBN -13:978-3-642-68699-3 e- ISBN -13 :978-3-642-68697-9 DOl: 10.1007/978-3-642-68697-9 This work is subject to copyright. [13.14] Pugsley, A. 23, 1944. Admittedly this approach can be adopted only when such information is available_ Expressing this problem in another way, it is important that the statistical analysis of data should be restricted to samples which are homogeneous (or more precisely - for which there is no evidence of non-homogeneity). The inte1 2 grals in (10.12) are much more convenient than the integrals in (10.11) because they only involve rates of upcrossing of the processes {Xl} and {X 2 } and the corresponding density functions. The only sensible approach is to synthesise the probability distribution of X from a knowledge of the component sources of uncertainty (as in equation (3.33)). K.:Probabilistic Theory of Structural Dynamics. For further information the reader is referred to Aitchison and Dunsmore [3.1]. Failure of a particular component may eventually occur for one of these reasons, or it may be caused indirectly as a result of the deterioration of some other part of the system. Vol. This extension is important because in practical applications basic variables will often be cor· related and because correlation sometimes affects the value of {3 appreciably. mean and standard deviation of the approximate normal distribution. [6.8] Thoft-Christensen, P.: Some Experience from Application of Optimization Technique in Structural Reliability. Turkstra, C. Characteristic values: The uncertainty in most permanent loads is small and for this reason it is customary to use the mean or nominal values of permanent loads in most design calculations. on Wind Effects on Buildings and Structures, Ottawa, Sept. The safety and serviceability of practical structures are achieved by the use of suitable partial coefficients in design, together with appropriate control measures. Institution of Civil Engineers, London, Vol. [4.5] Cornell, C. In many respects the first step is both the most important and the most difficult to decide upon in practice. 57 3.5 MODELLING OF LOAD VARIABLES· MODEL SELECTION Variable loads: For continuous time-varying loads which can be uniquely described by a single quantity X (e.g. a magnitude), one can define a number of different but related probability distribution functions. The same is true for resistance variables. The other data used in example 5.6 are unchanged. BIBLIOGRAPHY 93 Exercise 5.4. Consider the same elastic beam as in exercise 5.3, but with PM assumed unF known. The words »perfectly brittle» should be understood in a broad sense. +qm fXlAl n Bm (x) , I m 1 qj =1 (3.34) qj being the probability that the bar is of diameter j. , Xn) is chosen, and the failure surface is expressed in the corresponding x-coordinate system. 1 Then fX~ (x)dx = constant X probability that (i - 1) values of X fall below x 1 X probability that (n - i) values of X fall above x X probability that 1 value of X lies in the range x to (x+ dx) = cF~-l (x)(l- Fx(x))n-ifx(x)dx (3.1) where c = (i ~ (~) ~; ~ i) ! = the number of ways of choosing (i - 1) values less than x, together with (n - i) values greater than x (3.2) Thus (3.3) This can be shown to be equal to +-_____ ~ ~~ _____ x dx Figure 3.1 _=~ __ ~x 39 3.2 STATISTICAL THEORY OF EXTREMES _ n(n-1)! FX~(Y)-(i-1)!(n-i)! [(FXiy»i_(n1-i) (FX(y»i+l (n-i)(i+1) + 2 X (Fx (y»i+2 _ (n-i) (n-i) (FX(y»n (i+2) ... Compare with exercise 2.5. Exercise 2.11. [5.3] Ditlevsen, 0.: Fundamentals of Second Moment Structural Reliability Theory. 62, Feb. VII Thanks are due to our respective colleagues in Aalborg and London for their helpful comments and contributions and in particular to Mrs. Whether the weights of individual structural elements can also be assumed to be normally distributed depends on the nature of the processes controlling their manufacture. Hence the median value of 8, = exp(4.3928) = 80.83 mm. The design value e d of the strength of a material is obtained from the specified or characteristic strength as follows = e sp e d)' m (11.12) where), m is a partial coefficient on strength. In general, one can estimate from test results or records the parameters of the distribution of the instantaneous values of load or of the strength of individual components, and from this information the aim is to derive the distribution for the smallest or largest values. Title. M is the safety margin sometimes referred to as the failure indicator. SPECTRAL ANALYSIS OF WAVE FORCES Equation (B.ll) consists of a set of uncoupled modal equations which can be used to obtain the structural response in each »normal» mode separately, treating each as a one degree of freedom system. The problem then is how best to use all this information. Should the nominal dimensions be increased or reduced, and if so by how much? and Squire, E. Volume 35,1981. Failures of type B occur mainly because of lack of knowledge. BIBLIOGRAPHY [3.1] Aitchison, J. Let us now define the quantity X as the random variable representing the yield stress of a particular grade of reinforcing steel irrespective of source and where »yield stress» is defined in a precise way_ We now wish to establish a suitable probability density function for X to use in further calculations. [13.4] Bosshard, W.: Structural Safety - A Matter of Decision and Control, IABSE Surveys S 9/79, IABSE Periodica 2/1979. Failures of type A arise because of gross errors in the calculation of design loads and/or loadcarrying capacities, and/or because of weaknesses which are introduced into the structure during construction. Part I: Material Variability. The structure is proportioned to carry a design load Qd = r Q qsP' where r Q is a partial coefficient on live load. 4.2 ELEMENTS OF CLASSICAL RELIABILITY THEORY Classical reliability theory was developed to predict such quantities as the expected life, or the expected failure rate, or the expected time between breakdowns of mechanical and electronic systems, given some test or failure data for the system and/or its components. It should not be thought that all these developments have been fully co-ordinated, or that all the changes to practical codes that have taken place are necessarily of great benefit. The main future use, how· 234 12. 2, Trondheim, 1976. 3.2.2 Normal extremes If a random variable is normally distributed with mean I1x with standard deviation aX the vari- able has a distribution function Fx (see (2.46» F (x)= X ,X \ • _ ~ 1 -:= V2rr 1 1 t-11 2 -exp(--(_-…X))dt aX 2 aX (3.7) If we are interested in the distribution of the maximum value of n identically distributed normal random variables with parameters I1x and ax this has a distribution function (3.S) It should be noted that Fxn is not normally distributed. More specifically, let two loads (or load effects) be modelled by stationary and independent stochastic processes {Xl (t), 0..:; t..:; T} and {X 2 (t), 0";; t..:; T}. 177 Chapter 11 APPLICATIONS TO STRUCTURAL CODES 11.1 INTRODUCTION Structural codes are documents which, by their very nature, are subject to periodic revision and amendment, but the decade 1970 - 80 was a time of marked activity in code development. Convergence to one of these points would result in a dangerous over-estimation of the reliability index and hence It{. Indeed many recent changes in structural codes have not been met with enthusiasm by practising engineers, often for good reason. 1st International Conference on the Behaviour of Offshore Structures, Vol. Bericht Nr. 59, Institut fUr Baustatik und Konstruktion, ETH, ZUrich,1976. These distributions are frequently used for representing the maxima of time-varying loads. The main features have been • the replacement of many simple design rules by more scientifically-based calculations derived from experimental and theoretical research, • the move towards limit state design - whereby the designer and/or code writer specifies the relevant performance requirements (limit states) for each structure explicitly; and where separate sets of calculations are required to check that the structure will not attain each limit state (at a given level of probability), • the replacement of single safety factors or load factors by sets of partial coefficients, • the improvement of rules for the treatment of combinations of loads and other actions, • the use of structural reliability theory in determining rational sets of partial coefficients, and • the preparation of model codes [11.7] for different types of structural materials and forms of construction; and steps towards international code harmonisation, particularly within the European Economic Community (EEC). These show that the failure probability is extremely sensitive to the variance of the extreme wind speed and to the mean value of the yield stress. Both branches have developed from classical reliability theory - a subject which evolved as a result of the increasing need for reliable electronic systems during the 1940's, initially for military applications. Publication S72:4. For simplicity, let Rand S be normally distributed and J..IR = 380 N/mm2 J..IS = 230 N/mm2 Then, from equations (4.26) and (4.27), fj = 3.02 and Pf = 1.3 X 10-3 • Assume now that a lower grade of steel is used in place of the correct grade, so that R is distributed with parameters J..IR = 280 N/mm2 giving fj' = 1.0 and ail. In chapter 5 it was mentioned that the reliability index as defined by equation (5.9) is not invariant with regard to the choice of failure function. Up,l.. Clearly, for non-linear failure functions, calculation of the reliability index {J = !lM laM on the basis of a linearization as (5.21) will depend on the choice of linearization point. 161 Chapter 10 LOAD COMBINATIONS 10.1 INTRODUCTION The modelling of load variables is treated briefly in section 3.5. It is stressed there that load variables and other actions are typically time·varying quantities which are best modelled as stochastic processes. For example, for the sum {X} of three independent processes {Xl}' {X 2} and {X 3 } one gets Vxm ~ Jr_ ~ ~."1 [v (x)fx + X (~- x) + Vx (x)fx + X (~- x) + Vx (x)fx + X (~- x)ldx 2 3 2 1 3 3 1 2 y (10.14) where the density functions fX;, + Xj are determined as usual by the convolution integral f x .+ x. It was the intention to make this book moderately self-contained and for this reason chapter 2 is devoted to the essential fundamentals of probability theory. They can be characterized as being probabilistic methods of analysis based on knowledge of the (joint) distribution of all basic variables. As might be expected a rather similar problem arises in treating combined loads within the framework of a deterministic level 1 code. See, for example, figures 3.4 and 3.10. C with terms 1 (B.6) where a·r ,I. With the same mean and standard deviation for P, E and I as in example 5.5 one gets a = 1.2825 (kNtl and u = 3.5499 kN. and a~. EXTENDED LEVEL 2 METHODS 10 kN IlQB = 11MB UQB = 1.4142 kN uMB = 11.1803 kNm = 75 kNm and In example 6.1 and exercise 6.1 random variables Q B and MB both depending on the same variables PI and P 2 are considered. 1967, Vol. However, a complete book would be required to cover this subject in depth. LOAD COMBINATIONS Only the sum of two independent processes has been treated above. and Penzien, J.: Response of Offshore Structures to Random Wave Loads.

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